Temporary Price Increase During Replenishment Lead Time

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Pricing is the most natural mechanism of revenue management. Firms use various forms of dynamic pricing, including personalized pricing, markdowns, promotions, coupons, discounts and clearance sales, to respond to market fluctuations and demand uncertainty. In this paper, we study a temporary price increase policy, as a form of dynamic pricing, for a non-perishable product, a practice used by several giant retailers such as Amazon, Walmart, and Apple. We develop a continuous review inventory model that allows for joint replenishment and pricing decisions, where the lead time is not zero: A replenishment decision controls supply, while a pricing decision controls demand. A manager exercises a temporary price increase to slow demand and avoid a stock-out situation while waiting for a shipment, which may not necessarily increase revenues. The problem is to solve for the optimal replenishment (when and how much to order) and pricing policy parameters (when and how much to raise the price) to maximize the long-run expected profit. We solve many numerical examples and perform extensive sensitivity analyses. Compared to a model that focuses on fixed pricing, our model brings an additional increase in profit of about 13%.
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1. Introduction

Matching supply and demand has always been one of the challenging issues faced by practitioners and supply chain researchers. Despite intense efforts, there is no clear-cut solution to a perfect match mainly due to the unavoidable variability in both demand and supply. Vitasek et al. (2003) suggest accounting for demand volume and variability in chronic supply-demand mismatch situations. In practice, firms always adjust the prices of their goods to please their customers. Firms use various forms of dynamic pricing, including personalized pricing, markdowns, promotions, coupons, discounts and clearance sales, to respond to market fluctuations and demand uncertainty. On the supply side, long lead times result in supply uncertainty. Firms across the U.S. and Europe have been increasingly looking to outsource their supply to non-domestic sources in an effort to lower their procurement costs, increase flexibility, and become more effective in global level (Fersht 2013). On the other side of the coin, freight may add four to six weeks to the delivery time (Quint and Shorten 2005), which increases supply uncertainty and risk for unsatisfied customers. According to Corsten and Gruen

(2004), only 15% of customers realizing stockouts will postpone the purchase to a later time when the item is back in stock.

Retail firms commit to order quantities well before the selling season and, in some cases, to stocking levels at stores. Thus, it is less costly for a retailing firm to change the price of an item than to ship items from one store to another to encounter a stockout situation. It costs an online retailer nothing to change prices. This flexibility is an advantage it has over a conventional retailer. Changing prices is a revenue management practice that allows a firm to control sales, where retailers have been researching pricing strategies to increase their revenues and subsequently profits. When it occurs, a firm usually responds by temporarily increasing the price of the product to slow demand and reduce the chances of running short. Many retail giants, such as Amazon, Walmart, and Apple, have this as a business practice.

A case study, conducted by a leading research software for Amazon sellers on real products sold on Amazon.com, indicates that if a seller on Amazon is low on inventory and stock levels will not hold out until the next shipment arrives, the seller can then raise prices on the remaining products to avoid going out of stock (Sellics 2016). This action, common among Amazon sellers, slows the demand and ensures that the seller will not go short before the next shipment arrives. The results of the study indicate that a seller incrementally raised the price of a specific product (from $19.9 to $49.9) over a period of 31 days when faced with low inventory level. Despite the drastic price increase, the product went out of stock and disappeared from Amazon's search results. After twenty-four days of being out of stock, the product was available again on Amazon at a retail price of $24.9.

Upstream Commerce, a retail intelligence that uses predictive and dynamic pricing solutions, analyzed the pricing behaviors of Amazon, Walmart, Best Buy, Macy's and Zappos over the holiday shopping seasons of 2011, 2012 and 2013. It found Amazon raised its prices for Cyber Monday when other retailers discounted theirs on Black Friday (Businesswire.com 2014, Retailewire.com 2014) and that, although faced by a surge in demand, Walmart's average prices were higher on both Black Friday and Cyber Monday (in 2011). The research study notes that Amazon's behavior on Cyber Monday was similar to Walmart's. It raises prices of many and less expensive products that are usually not restocked once they go out of stock.

Most online retailers that sell non-perishable goods temporary increase prices to slow demand and encounter the possibility of stockouts, which is a cheap approach that requires no effort. Bhargava *et al.* (2006) wrote, "short-lived price changes can be announced quite costlessly online". Baker *et al.* (2001) mention that the internet allows for the dynamic pricing of products, without ignoring the conditions of companies and the behavior of customers. For a detailed review of the impact of the internet on pricing strategies, the reader is referred to Chan *et al.* (2004).

The temporary price increase applies especially to seasonal and fad items, whose demand is often difficult to predict. For example, at Christmas, no one can accurately predict which toy will rank at the top of sales until the season is over. Because of long production times, it is not always possible to fulfill shortages quickly, which pushes retailers to increase the prices temporarily (Mochizuki 2017). For example, in the Christmas of 2006, respectively, Tickle-Me Elmo (a list price of $39.99) and Nintendo Wii (a list price of $539.93) were in such a short supply that sellers on eBay were asking $100 and $3000 for these items (Taylor 2006).

Chip prices more than doubled in the year before Apple announced its launching of the iPhone 8. The demand for this phone was expected to worsen the global squeeze on the supply chips. Clients for phone chip suppliers are accepting higher prices to make sure they get enough memory chips for their products (Lee 2017). The iPhone remains to be a critical source of demand given the huge sales volume and recent moves to increase storage capacity on the device. The surge in demand has affected the NAND market. Long (2017) notes that a shortage of NAND in 2017 will boost the prices of SSD (solid state drives), by about 9%, which are the standard for fast storage. Shortages of LCD screens required Apple to temporarily increase the prices of replacing them with iPhone models 5, 5C, and 5S (Chen 2015). Similarly, Parallax Inc., a provider of electronic hardware and software such as RFID systems announced on July 14, 2014, that while they were resolving an agreement with the intellectual property legal company representative on a patent right issue, they temporarily increased the prices of RFID tags to reduce customer demand. This price increase decreased demand for RFID by about 70%. After resolving the royalty issues, Parallax reduced RFID tag prices by 70 % (Gracey 2014).

Another example applies to the US car industry. U.S. dealers started realizing shortages for some Japanese vehicles in the months following the 2011 earthquake in Japan. According to Boudette (2014), to keep up with the supply shortage, Japanese vehicle prices have increased by an average of $400 per vehicle. In this regard, he wrote “new-car sales were running at a slower pace than before the quake..., but store profit was well above the previous year’s level and almost at its target level”.

The above examples are actual practices of temporary price increases adopted by many companies from different industries; e.g., retail, high tech, and automotive manufacturing, to name few. . The literature on dynamic pricing has focused solely on varying prices dynamically in the context of markdowns, promotions, discounts, clearance, personalized pricing, in the face of fluctuating demand. To the best of our knowledge, the Operations Management literature shows no work on the practice of temporary price increase during the lead time period to cut down on the lost sales caused by product shortages. Thus, this study is the first to address this topic.

We consider a continuous review inventory replenishment model for a single product. The model assumes a single product, fixed ordering cost and non-zero lead time. An order is triggered whenever inventory drops to a predetermined level. Shortages result in lost sales. During an order lead time, the firm increases the price temporarily to reduce the effect of an inventory shortage. After an order arrives, the firm charges back the original product price. This paper illustrates how a temporary price increase can be used to better match supply and demand in the case of supply shortages and would yield significant profit improvements. Our objective is to develop a model to determine the optimal inventory replenishment and pricing policy under a temporary price increase. We then compare the results to a fixed price policy to gain insights into the benefits that a temporary price increase brings to a company. Our results indicate that introducing a temporary price increase has negligible effects on the regular replenishment policy. Its significance becomes more pronounced when the reorder level is lower. Moreover, it yields a higher increase in profits for relatively slow moving and price sensitive products.

In our model, we define two price change trigger variables. The price change when the inventory during a replenishment lead time hits a trigger level in a specific time window. The rationale for a temporary price increase is to slow demand and buffer against supply shortage until a shipment is received. We formulate a long-run expected profit function for the continuous review problem with price increase applied during the lead time.

The simultaneous optimization of the inventory replenishment and pricing policies is analytically difficult to track with four dependent decision variables: the inventory replenishment decisions (when and how much to order), and the pricing policy decisions (when and how much to increase price). Thus, we conducted extensive numerical analyses because it was difficult for us to provide an analytical solution due to the untrackable mathematics. We observe that a temporary price increase during the lead time leads to considerable profit improvements which has valuable managerial implications, as this policy is easy to implement in practice. It consists of one price change only, as opposed to multiple price changes suggested in the literature. Despite its simplicity, it leads to significant profit improvements. Adopting a one price change only in which price switching happens from regular to high, and then back to regular, simplifies operations, is easily implemented, and occurs at a lower cost than adopting more complex policies.

The authors believe that this paper is the first to introduce a single price increase as a hedge against supply shortage, and the first in the dynamic pricing literature, in the context of markdowns and promotions, to consider a fixed cost and non-zero lead time. Unlike the work on dynamic pricing in the literature, this paper assumes continuous review with a fixed cost and non-zero lead time. The presence of a positive lead time significantly complicates the model. In this regard, we believe the contribution of this paper is significant and enriches the relevant literature. We also offer a simple policy that is easy to implement in practice and demonstrate the significant benefits from adopting it. When compared to the conventional model, the approximate (exact) solution improved profits, on average, by 9% (13%).

This paper is organized as follows. The related literature is reviewed in §2. In §3, we develop the objective function by calculating the expected revenue, costs, and cycle time conditioned on the price change probability. Section 4 reports numerical results and analyses and discusses them. It also provides some managerial insights. Moreover, we show how simplified models can be utilized to obtain significant benefits with a price change. We present our conclusions and future directions in §5.

2. Literature Review

Our study is related to two distinct streams of research, joint pricing and inventory replenishment, and emergency orders. The literature on the joint optimization of pricing and inventory decisions is quite rich and has been referred to repeatedly in studies on revenue or yield management (Gallego and Van Ryzin 1994, Elmaghraby and Keskinocak 2003).

In the joint pricing and inventory replenishment research stream, the earliest work is that of Whitin (1955) which models price-dependent demand in inventory replenishment problem and determines the price endogenously. For replenishable products, pricing decisions are made either after every customer arrival or every replenishment period. Federgruen and Heching (1999) and Chen and Simchi-Levi (2004) model periodic review systems where a firm determines in each period its joint inventory and pricing policy before demand has been realized. Chen and Simchi-Levi (2006) and Chao and Zhou (2006) investigate joint dynamic pricing and inventory policies for a continuous review system. They recommend altering the price after every demand arrival. Chen *et al.* (2010) consider a continuous review inventory model that allows for multiple price changes at different inventory levels in between two replenishments. The studies surveyed above assume the lead time is zero, assuming otherwise remains as a challenging problem to solve (Chen and Simchi-Levi 2006). Our work contributes to the dynamic pricing literature in that it develops a model with a non-zero lead time. For complete reviews on the joint dynamic pricing and inventory control problems, see Elmaghraby and Keskinocak (2003), Chan *et al.* (2004), Yano and Gilbert (2005), and Gimpl-Heersink (2008).

Although the research on joint control of inventory and pricing uncovers the benefits of dynamic pricing, it shows that few price changes are enough to attain the objective. Chen *et al.* (2010) and Gayon *et al.* (2009) show that two price changes are enough to reap the benefits of dynamic pricing. Gallego and van Ryzin (1994) show that single price change policies asymptotically reach the optimal prices with an increase in sales. Similarly, we consider a single price change during replenishment lead time and investigate its benefits. Moreover, for a continuous review, zero lead time, and backlogging model, Chao and Zhou (2006) show that as the inventory level decreases (as long as it is non-negative), the optimal price to charge increases. So, when a single price change is allowed, it usually results in a price surge. Similarly, we consider a price increase during the lead time while the inventory level is decreasing.

Our study is also related to the research works that consider emergency shipments between subsequent regular replenishment orders. There are various methods to deal with emergency shipments in the literature. For example, last-minute emergency orders from a single (Moinzadeh and Nahmias 1988) or multiple suppliers for higher unit prices (Tomlin 2006), order splitting (Thomas and Tyworth 2006) and reduce the order lead time (faster production and delivery) at an additional cost (Duran *et al.* 2004). Moreover, when the regular replenishment from the manufacturer is not available to satisfy immediate demand, transshipment of goods within the same echelon can be initiated, such as trading vehicles among dealers when the shipment from the factory is still due (Çömez *et al.* 2012).

This paper is closely related to a few earlier works that study the changing demand pattern between replenishments when inventory is low. Xu *et al.* (2011) assume that a retailer stops selling an item when its inventory level is critically low. They use this policy to encounter shortage costs. Cheung (1998) models a continuous review inventory system where a retailer offers a price discount to customers willing to backorder their demand until the next replenishment. Not accepting the offer results in a lost sale. DeCroix and Arreola-Risa (1998) introduce offering compensation to convince customers to backorder in an (s, S) inventory model. Ding *et al.* (2007) consider multiple customer classes, each has its own price discount for back-ordering demand. Using a deterministic EOQ model, Bhargava *et al.* (2006) determine the lengths of the in-stock and stockout periods and the item's price. Drake and Pentico (2011) modify an EOQ by making the percentage dependent on the size of the discount. They investigate the optimality of offering a discount and its size. In this paper, we consider only one price change during a replenishment cycle. Chen *et al.* (2010) show that a single price change (i.e. two prices) is enough to reach the most benefit. A firm may avoid emergency orders if it exercises a price change. Moreover, the period of time during which the firm needs an emergency action may not be long enough to accommodate multiple price changes. Excessive price changes may be costly depending on the sales channel. It may be, among many, updating the price tags and the associated web pages. Gayon *et al.* (2009) discuss that frequent price changes are perceived negatively by customers.

3. Model

We study a continuous-review inventory replenishment model of a firm that is managed by the well-known (Q, R) policy, where R is the reorder point and Q the order size. Thus, whenever the inventory level hits R, an order of size Q is placed. The system goes through one replenishment cycle between the placement of two successive orders, where the inventory level R renews itself(goes from R back to R). The notation used is listed in Table 1 below.

In a continuous review inventory model, when the unmet demand is lost, the number of outstanding orders is the largest integer less than or equal to $(Q + R)/Q = 1 + R/Q$ (Tekin et al. 2001). Then, if we assume that $R < Q$, it is always true that there is only a single order outstanding, which is an assumption that has been extensively used in the literature; e.g., Hadley and Whitin (1963), Archibald (1981), Moinzadeh and Nahmias (1988), Cheung (1998), Johansen and Thorstenson (1998), Tekin et al. (2001), and Duran *et al.* (2004).

During a replenishment cycle, the firm faces a price-sensitive, stochastic demand. Let $D(p, t)$ be the random demand at time t for price p. $D(p, t)$ has a probability density function $f(\cdot, p, t)$, cumulative distribution function $F(\cdot, p, t)$, and mean $\mu(p)$. We assume that demand in consecutive time periods is independent. Each item kept in inventory for a unit of time incurs a unit cost of h.

Table 1: Notation

Parameters	
K	Fixed ordering cost per order
c	Procurement cost per unit
h	Inventory holding cost per unit per unit-time
b	Shortage cost per unit demand lost
L	Order lead time
p_1	Regular unit selling price
p_2	Temporary unit selling price during the lead time, where $p_2 > p_1$
$\mu(p)$	Mean demand rate per unit-time as a function of price p
$D(p,t)$	Demand at time t for price p with a probability density function $f(\cdot,p,t)$, a cumulative distribution function $F(\cdot,p,t)$, and a mean $\mu(p)$
Variables	
Q	Order quantity
R	Reorder point
r	Inventory level that triggers a temporary price increase from p_1 to p_2, where $r < R$
T	Time window within which a price change can be made after an order release
Other Variables	
τ	Time interval during which the inventory level drops from R to r with probability density function $g(t)$ and cumulative distribution function $G(t)$
θ	Probability of utilizing a price change during a cycle, i.e. $\theta = P(\tau \le T)$.

If demand at a point in time exceeds the inventory on-hand, then unmet demand is lost at a unit cost of b. The lost sales cost may include the loss of customer goodwill, lost future sales, as well as the compensations that would be offered to unsatisfied customer (Petruzzi and Dada 1999 and Serel 2009).

In the (Q, R) inventory model with at most one outstanding order at any given time, lost sales are realized during the order lead time L, which starts right after the inventory level hits R and until a lot of size Q is delivered. The firm charges a price p_1 when the inventory level is higher than R. During the lead time, if the inventory level reaches r, $0 \le r \le R$, the initial price p_1 is increased to p_2 until the end of the lead time, where r is the inventory level that triggers the price change. In our model, r is a decision variable to be determined along with the other decision variables R and Q. When the order is received at the end of the lead time period L, the regular market price p_1 is charged. For a price change to be rational, selling prices and corresponding demand rates should satisfy $p_1\mu(p_1) > p_2\mu(p_2)$ when $p_1 < p_2$. Otherwise the firm would always charge p_2, i.e., there would be no incentive to charge a lower price p_1 (Feng and Gallego 1995). The inventory dynamics are illustrated in Figure 3.

The time period during which a temporary price increase is exercised, is restricted to the first T time units after an order is released, T being a decision variable that satisfies $T \le L$. The reason is that if the inventory level hits r late during the lead time, then for the remaining time until the next order arrives, the remaining inventory r may be enough, which does not necessitate a price change. On the other hand, if the inventory level hits r earlier during the lead time, then the likelihood of realizing stock-outs later during the lead time is higher, which might be prevented by increasing the unit selling price to p_2, thus decreasing the demand rate.

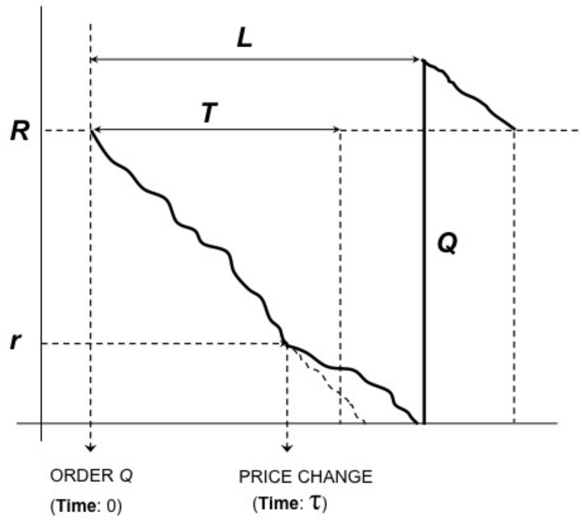

Figure 1: Inventory dynamics with a temporary price increase from p_1 to p_2.

One could conjecture that the time point to change the price depends on both the inventory on-hand and the time remaining to receive the next order. In their finite horizon perishable inventory model, Feng and Gallego (1995) show the optimality of time thresholds to find the best time for a single price change, either high to low or low to high. Cheung (1998) use a similar time limit for offering discounts to customers who will wait for delayed orders. In our infinite horizon model, such a time threshold policy occurs during the replenishment lead time.

Let the time when inventory level is R set as zero. Then let τ to denote the time when the inventory level hits r, $\tau \le T$. After an order is placed and during the interval $[0, T]$, if the inventory level reaches r, price p_2 is charged for the remaining time period $L - \tau$. If $\tau > T$, no price change is made and the regular unit price p_1 is charged for the whole replenishment cycle. Since the following events are equivalent, $\{\tau \le t\} = \{D(p_1, t) \ge R - r\}$, the cumulative distribution function $G(t)$ and the probability density function $g(t)$ of τ can be written as follows

$$G(t) = P\{D(p_1, t) \ge R - r\} = \int_{R-r}^{\infty} f(x, p_1, t)dx,$$

$$g(t) = \frac{d}{dt}[\int_{R-r}^{\infty} f(x, p_1, t)dx].$$

The firm's objective is to maximize the long-run average profit per unit-time, with the decision variables being (Q, R, r, T). To obtain the long-run average profit, we next calculate the expected revenue, the expected average number of units held, the expected number of lost sales, and the expected cycle time.

Expected Revenue Per Cycle

The expected revenue for a cycle, $E[RV]$, is obtained by conditioning on τ, the time when the inventory level hits r.

$$E[RV] = E[RV, \tau > T] + E[RV, \tau \leq T]$$

If $\tau > T$, there will be no price change, and p_1 is charged throughout the cycle. In this case, the total revenue is as follows

$$E[RV, \tau > T] = \int_T^\infty p_1 Q g(t) dt. \tag{1}$$

If $\tau \leq T$, p_2 is charged during the time interval (τ, L) while p_1 is charged for the remaining time in the cycle, i.e., between the reorder point (time zero) and τ and also between the order arrival and the next reorder point.

$$E[RV, \tau \leq T] = \int_0^T E[RV|\tau = t] g(t) dt. \tag{2}$$

The conditional revenue E$[RV|\tau = t]$ in (2) depends on whether or not there is a stock-out until the next replenishment.

$$[RV|\tau = t] = \begin{cases} p_1 Q + (p_2 - p_1) D(p_2, L-t), & \text{if } D(p_2, L-t) < r \\ p_1 Q + (p_2 - p_1) r, & \text{if } D(p_2, L-t) \geq r. \end{cases}$$

Then, we get the following revenue expressions

$$E[RV|\tau = t] = p_1 Q + \int_0^r (p_2 - p_1) x f(x, p_2, L-t) dx + \int_r^\infty (p_2 - p_1) r f(x, p_2, L-t) dx$$

$$E[RV, \tau \leq T] = \int_0^T \left(p_1 Q + (p_2 - p_1) \int_0^r x f(x, p_2, L-t) dx + (p_2 - p_1) r \bar{F}(r, p_2, L-t) \right) g(t) dt. \tag{3}$$

Summing up (1) and (3), the total expected revenue is obtained as

$$E[RV] = p_1 Q + (p_2 - p_1) \int_0^T \Big(\int_0^r x f(x, p_2, L-t) dx + r \bar{F}(r, p_2, L-t) \Big) g(t) dt,$$

where $\bar{F} = 1 - F$ denotes the complement of the cumulative distribution function F.

Expected Lost Sales per Cycle

The expected number of lost sales per cycle, $E[LS]$ is also calculated by conditioning on the time τ.

$$E[LS] = E[LS, \tau > T] + E[LS, \tau \leq T].$$

When there is no price change within a cycle, the event $\{\tau > T\}$ can be replaced by $\{D(p_1, T) < R - r\}$, as they are equivalent.

$$\begin{aligned} E[LS, \tau > T] &= E[LS, D(p_1, T) < R - r] = \int_0^{R-r} E[LS|D(p_1, T) = x] f(x, p_1, T) dx \\ &= \int_0^{R-r} E[D(p_1, L) - R|D(p_1, T) = x]^+ f(x, p_1, T) dx \\ &= \int_0^{R-r} \int_{R-x}^\infty (x + y - R) f(y, p_1, L-T) f(x, p_1, T) dy dx. \end{aligned}$$

When the demand within period T and after the placement of an order is at least $R-r$, a temporary price change is exercised . In this case, the expected number of lost sales per cycle is

$$\begin{aligned} E[LS, \tau \leq T] &= \int_0^T E[LS|\tau = t]g(t)dt = \int_0^T E[D(p_2, L-\tau) - r|\tau = t]^+ g(t)dt \\ &= \int_0^T \int_r^\infty (x-r)f(x, p_2, L-t)g(t)dxdt. \end{aligned}$$

Then the expected number of lost sales per cycle is obtained as follows

$$E[LS] = \int_0^{R-r} \int_{R-x}^\infty (x+y-R)f(y, p_1, L-T)f(x, p_1, T)dydx + \int_0^T \int_r^\infty (x-r)f(x, p_2, L-t)g(t)dxdt.$$

Expected Cycle Time

In a continuous review inventory model with random demand, the exact change in inventory levels is different in each cycle, whose length is a random variable. In a simple (Q, R) model with backorders, the expected cycle length is $Q/\mu(p)$, where Q is the total demand received per cycle and $\mu(p)$ is the expected demand per unit-time. When lost sales are considered, the expected cycle time is increased to $(Q+E[LS])/\mu(p)$, where E$[LS]$ is the expected lost sales per cycle, as the actual demand observed includes the lost part $E[LS]$.

In our model, the demand rate changes over a cycle if a price change is made. Thus, the expected cycle length is conditioned on the possibility of a price change occurring during a cycle.

$$E[CT] = E[CT, \tau > T] + E[CT, \tau \leq T].$$

Noting that the event $\{\tau > T\}$ is equivalent to $\{D(p_1, T) < R-r\}$, the cycle time expression when there is no price change is obtained as follows

$$\begin{aligned} &E[CT, D(p_1, T) < R-r] \\ &= \int_0^{R-r} E[CT|D(p_1, T) = x]f(x, p_1, T)dx \\ &= \int_0^{R-r} \Big(L + \int_0^{R-x} \frac{Q-x-y}{\mu(p_1)} f(y, p_1, L-T)dy + \frac{Q-R}{\mu(p_1)}\bar{F}(R-x, p_1, L-T)\Big) f(x, p_1, T)dx. \end{aligned}$$

If a price change is made, then the expected cycle time is

$$E[CT, \tau \leq T] = \int_0^T \Big(L + \int_0^r \frac{Q+r-x-R}{\mu(p_1)} f(x, p_2, L-t)dx + \frac{Q-R}{\mu(p_1)}\bar{F}(r, p_2, L-t)\Big) g(t)dt.$$

Then the expected cycle time is written as follows

$$\begin{aligned} E[CT] &= L + \int_0^{R-r} \Big(\int_0^{R-x} \frac{Q-x-y}{\mu(p_1)} f(y, p_1, L-T)dy + \frac{Q-R}{\mu(p_1)}\bar{F}(R-x, p_1, L-T)\Big) f(x, p_1, T)dx \\ &\quad + \int_0^T \Big(\int_0^r \frac{Q+r-x-R}{\mu(p_1)} f(x, p_2, L-t)dx + \frac{Q-R}{\mu(p_1)}\bar{F}(r, p_2, L-t)\Big) g(t)dt. \end{aligned}$$

Expected Average Inventory per Cycle

To calculate the expected on-hand inventory, following Hadley and Whitin (1963), we assume that for a (Q, R) model, the stock-out probability is sufficiently small. Such an assumption is reasonable in our case since a temporary price increase helps prevent or decrease stock-outs. Moreover, Lau and Lau (2002) show that the method that Hadley-Whitin used to compute average inventory is quite robust and often more accurate than alternative ones suggested in the literature. The exact computation of inventory costs is complicated especially for lost sales model relative to a backorder model. While the inventory position is uniformly distributed between R and $R+Q$ and independent of the current inventory state in a backorder model, it is not the case when demand is lost. Thus, approximate calculation of Hadley and Whitin (1963) is often preferred in the literature (Moinzadeh and Nahmias 1988, Cheung 1998, Johansen and Thorstenson 1998, Tekin *et al.* 2001, and Duran *et al.* 2004).

We calculate the expected average inventory by conditioning on the occurrence of a price change. First, we define the probability of a price change during a cycle, θ, such as

$$\theta = P(\tau \leq T) = P(D(p_1, T) \geq R - r) = \int_{R-r}^{\infty} f(x, p_1, T)dx,$$

where $(1 - \theta)$ is the probability of no-price change within a replenishment cycle.

The average demand rate with price p_1 during the lead time L can be calculated by conditioning on the given information that a price change is made or not. Define λ_1 as the average demand rate with price p_1 within time period $(0, T)$ given that no price change is made during a cycle.

$$\begin{aligned} \lambda_1 &= E[D(p_1, T)|D(p_1, T) < R - r])/T \\ &= E[D(p_1, T), D(p_1, T) < R - r]/(TP(D(p_1, T) < R - r)) \\ &= \int_0^{R-r} xf(x, p_1, T)dx/(T(1 - \theta)). \end{aligned}$$

When a price change is made, the average demand rate with price p_1 during the period T is denoted by λ_2 and obtained as follows.

$$\begin{aligned} \lambda_2 &= E[D(p_1, T)|D(p_1, T) \geq R - r]/T \\ &= E[D(p_1, T), D(p_1, T) \geq R - r]/(T\theta) \\ &= \int_{R-r}^{\infty} xf(x, p_1, T)dx/(T\theta). \end{aligned}$$

By definition, the following equality should also hold.

$$(1 - \theta) * \lambda_1 + \theta * \lambda_2 = \mu(p_1).$$

Then, the expected average inventory can be written by calculating the areas under the expected inventory levels shown in Figures 2 and 3. $E[OH|\tau > T]$ and $E[OH|\tau \leq T]$ are the expected average

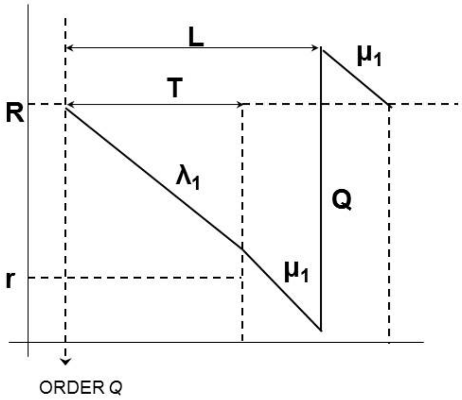

Figure 2: Expected inventory levels when no price change is exercised.

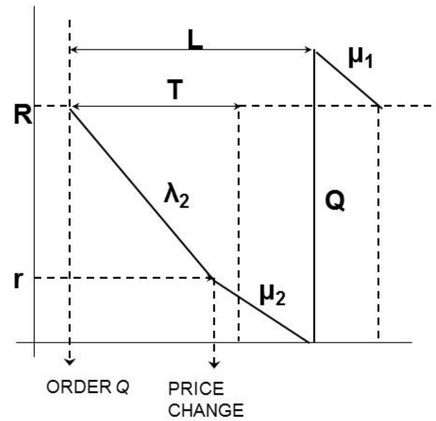

Figure 3: Expected inventory levels when a price change is exercised.

inventories when there is no price change and when there is price change, respectively.

$$\begin{aligned} E[OH|\tau > T] &= RT - Q(L-T) + \lambda_1 T(\frac{1}{\mu(p_1)}(\frac{\lambda_1 T}{2} - Q - R) - \frac{T}{2}) + \frac{Q}{\mu(p_1)}(\frac{Q}{2} + R) \\ E[OH|\tau \le T] &= \frac{R^2 - r^2}{2}(\frac{1}{\lambda_2} - \frac{1}{\mu(p_1)}) + \frac{Q}{\mu(p_1)}(\frac{Q}{2} + r) + \frac{\mu(p_2)}{2}(L - \frac{R-r}{\lambda_2})^2(\frac{\mu(p_2)}{\mu(p_1)} - 1) \\ &\quad +(L - \frac{R-r}{\lambda_2})(r - \frac{r\mu(p_2)}{\mu(p_1)} - \frac{Q\mu(p_2)}{\mu(p_1)}). \end{aligned}$$

Note that during the calculation of expected inventory levels $E[OH|\tau > T]$ and $E[OH|\tau \le T]$ following Figures 2 and 3, the inventory level right before an order arrives is not positively restricted, which can be correct when backordering is available. However, inventory level is always non-negative when unmet demand is lost. Therefore, consistent with Hadley and Whitin (1963), the expected inventory levels should be adjusted by adding the term $E[SL]E[CT]$. This term accounts for an increase in the inventory level resulting from losing a sale when back-ordering is not allowed. So, the unconditional expected amount of inventory held is obtained by conditioning on the probability of a price increase and also adjusting for the expected lost sales.

$$E[OH] = (1-\theta)E[OH|\tau > T] + \theta E[OH|\tau \le T] + E[SL]E[CT]. \tag{4}$$

With the calculation of the expected revenue, expected number of lost sales, and the expected number of average units held during a cycle, the expected profit per cycle is obtained such that

$$\pi(Q, R, r, T) = E[RV] - K - cQ - hE[OH] - bE[SL].$$

Given that the expected cycle time is also a function of the decision variables, the objective function is to maximize the expected profit per unit-time.

$$\max_{Q,R,r,T} \quad \Pi(Q,R,r,T) = \pi(Q,R,r,T)/E[CT]. \tag{5}$$

The analytical analysis of the objective function to derive closed-form expressions for the optimal values of the decision variables is quite complex. Therefore, we focus next on numerically analyzing the model and gaining managerial insights. In §4, the optimal long-run profits and optimal decision variables are obtained through complete enumeration of variables.

4. Numerical Analysis

Numerical analyses are conducted to gain insights into the solution of the model, the sensitivity of the solution, and the simplified model. We compare the temporary price increase model to a replenishment model with no price change and illustrate the benefits our model brings; i.e., increasing the total expected profit and managerial implications and insights.

To conduct the numerical analyses, we need to define the demand function in more detail. $D(p,t)$ is demand and random during time interval t for price p, which is an independent variable. An additive form is used to model the random component of $D(p,t)$ as in Gimpl-Heersink (2008), Jadidi *et al.* (2017), Petruzzi and Dada (1999), and Ray *et al.* (2005). Particularly, demand rate is defined as $D(p,1) = y(p) + \epsilon$, where $y(p)$ is a decreasing function of p and ϵ is a non-negative random portion. Particularly, we use $y(p) = \alpha - \beta p$, where $\alpha > 0$ and β is the price sensitivity coefficient. The price is restricted to the range $[c, \alpha/\beta]$ to prevent negative $y(p)$. Random demand portion of the demand ϵ is assumed to follow a Poisson distribution with mean μ, for ease of computation as in many studies such as Chen *et al.* (2010) and Chen and Simchi-Levi (2006). Then, the demand during a time interval t, $D(p,t)$, is the sum of $(\alpha - \beta p)t$ and the random portion, which follows a Poisson distribution with mean μt. The density function of total demand during period t is

$$f(x,p,t) = \frac{e^{-\mu t}(\mu t)^{(x-(\alpha-\beta p)t)}}{(x-(\alpha-\beta p)t)!} \quad \text{for} \quad (x-(\alpha-\beta p)t) \in \{0,1,2,...\}.$$

Accordingly, the probability density function of τ, which is the time for the inventory level to drop from R to r, is as follows

$$g(t)e = \frac{((\alpha-\beta p)t+\mu)e^{-\mu t}(\mu t)^{R-r-(\alpha-\beta p)t-1}}{(R-r-(\alpha-\beta p)t-1)!}.$$

Correct choice of p_1 and p_2 is important for the evaluation of the results. For an arbitrary value of p_1, if the introduction of an arbitrary price p_2, $p_2 > p_1$, leads to a higher total profit over the model with no price change, then this increase cannot be solely be associated with the use of a temporary price increase. Rather, p_2 can be a better choice than p_1 even for a constant price (Q,R) model. Thus, for each problem instance, we first obtain the optimal price for a model with no price change denoted by p', where a single optimal price p' is charged throughout the horizon. The price p' is obtained by modeling and solving a joint inventory and fixed pricing problem such that the long-run total profit is maximized over (Q, R, p) by benefiting from the optimality properties shown in Çömez and Kiessling (2012). Then, p_1 is set equal to the p' while solving the temporary price change model.

The optimal profit (5) is evaluated by first setting $p_2 = p_1$, i.e. no price change is allowed. The resulting optimal order quantity, reorder point, and expected profit per unit-time are denoted respectively by Q^0, R^0, and Π^0. When there is no price change, the price change trigger-level and the price change window, respectively r and T, are irrelevant. Next, (5) is evaluated for a price increase model with the increased price p_2 set to various feasible values for testing. Given that α/β is the upper bound for the chargeable price, for each problem instance, we test for $p_2 \in \{1.05p_1, 1.1p_1, 1.15p_1, ..., \alpha/\beta\}$.

To determine the problem parameters, we benefit from the values used by Chen *et al.* (2010) and Chen and Simchi-Levi (2006). The base problem setting has $K = 55$, $c = 10$, $L = 1$, $h = 1.5$, and $b = 30$. The base demand setting is $\alpha = 40$, $\beta = 2.25$, and $\mu = 5$. By changing each parameter at a time while keeping the others fixed, a total of 29 problem instances are generated. Each problem instance is run with various p_2 values, where $p_2 \in \{1.05p_1, 1.1p_1, 1.15p_1, ..., \alpha/\beta\}$. Thus, a total of 78 problem instances are reported in Tables 2, 3, and 4. In each problem setting, only the parameter that is changed with respect to the base setting is reported.

First, using the developed model, we show that a temporary price increase increases the expected profit and investigate its sensitivity to changes in the system's parameters. Second, to demonstrate the usefulness of a simplified version of the model, we set the price increase period to L, thus reducing the set of decision variables to (Q, R, r). Third, we propose another simplification by solving a two-stage problem. In stage 1, we obtain Q and R for no price change. In stage 2, we use Q and R to determine the price change trigger-level r. Fourth and last, we report the results of extensive numerical analyses run with randomly generated problem settings.

4.1 Benefits of temporary price change over a fixed price system

In this section, the magnitude of the change in expected profit as a result of a temporary price increase policy and the sensitivity of the model to system parameters are evaluated through 78 problem instances defined above. For each problem instance, first p_1 is obtained by solving a joint inventory and single price problem (Q, R, p), then the optimal profit (5) is evaluated by setting $p_2 = p_1$ and resulting optimal decision variables are Q^0, R^0, and Π^0. Next, (5) is evaluated for a price increase model for $p_2 \in \{1.05p_1, 1.1p_1, 1.15p_1, ..., \alpha/\beta\}$ where α/β is the upper bound for the chargeable price, for each problem instance. According to this range, for each problem setting, the number of tested p_2 values ranges between two and four. For example, in Table 2, for the first problem set, $p_1 = 16.12$ and p_2 has three feasible values according to our setting, where the highest value of p_2 tested is $\alpha/\beta = 40/2.25 = 17.78$. For the second problem set, $p_1 = 15.81$ and p_2 has set to two different values, where its highest value is $\alpha/\beta = 38/2.25 = 16.89$. The optimal decision variables are denoted by Q^*, R^*, r^*, and T^* with the the resulting expected profit per unit-time Π^*. Optimality search is done over integer values for Q, R, and r. For T, the search is done in increments of 0.1. The percent increase in the expected profit by utilizing a price change is denoted by $\Delta\Pi^* = (\Pi^*(Q^*, R^*, r^*, T^*) - \Pi^0(Q^0, R^0))/\Pi^0(Q^0, R^0) * 100$. Table 2 reports the results.

From Table 2, in all problem instances, we observe an increase in the expected profit when a price increase is allowed. The profit increases when it is possible to increase p_2. Increasing p_2 decreases r and increases T. This shows that the increase in the altered price is balanced by lowering the price change trigger level to limit the high price to a lower amount of product sales. It is observed that when a price change is allowed, the reorder level is slightly lowered ($R^0 \geq R^*$), but the change in order quantity can be in either direction ($Q^0 \geq Q^*$ or $Q^0 \leq Q^*$), while all observed changes are mostly negligible. The effect of each altered parameter has the same direction on inventory replenishment variables Q and R both in the model without a price change and with the price change. So, in below discussions we use notations Q and R, respectively, to refer to optimal order size and reorder level both in no price change and price change models.

Our sensitivity analysis indicates the following insights. It is observed that the benefit of a price change, measured by $\Delta\Pi^*$, in general increases with an increase in β, K, c, h, or b and with a decrease in α or μ. Particularly, when α or μ is low, or K, c, or h is high, the expected profit under no price change decreases significantly, resulting in a higher percentage increase by the price change.

$\Delta\Pi^*$ increases when the optimal reorder level R decreases with the change in the parameter. A valuable insight is that the price change becomes more important when lower levels of reorder level, i.e. lower inventory, are maintained, and used more effectively. For example, both procurement c and holding cost per unit h lead to a higher p_1 to increase profit margin, resulting in lower Q and R because of decreased demand rate, and so an increase in $\Delta\Pi^*$. Accordingly, $\Delta\Pi^*$ decreases while the optimal reorder level increases, making it less likely to benefit from a price change, as the possibility of inventory level hitting r decreases.

Except for K and b, an altered parameter affects Q and R in the same direction. With a higher fixed ordering cost K, while the order quantity Q increases, the reorder point R decreases. The increase in $\Delta\Pi^*$ is again because of the better use of reduced inventory level during the lead time. Our results on the effects of K are consistent with those in Chen *et al.* (2006) that study joint pricing and replenishment problem for periodic-review systems. It is surprising to observe that the optimal replenishment policy is almost insensitive to changes in the shortage cost b with a slight increase in R^*. So, as b increases, the lead time inventory level R and accordingly expected amount of prevented loss sales do not change much. On the other hand, the total value of prevented loss sales increases, so the benefit of exercising a price increase during lead time is more pronounced.

An increase in α or μ increases the expected demand rate, which increases p_1, the order quantity, and the reorder point. In such a case, the improvement in profits expected from a price change becomes less apparent. Because, with the higher R, the probability of a stock-out decreases, as well as the need for a price change. On the other hand, β, the price sensitivity coefficient, has a negative effect on p_1, Q, and R, and accordingly a positive effect on $\Delta\Pi^*$. Managerially, these results can be interpreted such that a price change can be more effective to increase the profit for relatively slow moving and price sensitive products. Besides, practically price change is relatively easier in implementation when demand arrivals are more dispersed in time.

As the new price p_2 increases, while we observe a higher improvement in profit, the price change trigger-level r^* decreases. Moreover, the optimal price change window T^* mostly increases with p_2 and in all instances $T^* \geq 0.5$. This result motivates us to introduce a simpler model, where the price change time window is exogenously set to the lead time and evaluated in the next subsection.

Table 2: Benefit of temporary price change over a fixed price system. Base setting has $\alpha = 40$, $\beta = 2.25$, $\mu = 5$, $K = 55$, $c = 10$, $L = 1$, $h = 1.5$, and $b = 30$.

p_1	(Q^0, R^0)	Changed Parameter	p_2	(Q^*, R^*, r^*, T^*)	$\Delta\Pi^*$ (%)	p_1	(Q^0, R^0)	Changed Parameter	p_2	(Q^*, R^*, r^*, T^*)	$\Delta\Pi^*$ (%)
16.12	(27,11)	-	16.93	(26,10,5,0.6)	7.1	16.01	(25,11)	$K = 45$	16.81	(24,11,5,0.6)	4.3
			17.74	(26,10,3,0.8)	13.1				17.61	(24,10,3,0.8)	8.3
			17.78	(26,10,2,0.9)	13.5				17.78	(24,10,3,0.8)	9.5
15.81	(25,9)	$\alpha = 38$	16.60	(25,8,5,1)	29.1	16.27	(28,11)	$K = 65$	17.09	(28,9,6,0.8)	12.3
			16.89	(24,8,5,1)	40.9				17.78	(28,9,3,0.8)	20.7
16.50	(28,12)	$\alpha = 42$	17.32	(28,12,4,0.7)	3.4	16.38	(30,10)	$K = 75$	17.20	(30,9,5,1)	20.3
			18.15	(28,11,4,0.7)	6.4				17.78	(29,9,3,0.8)	31.6
			18.67	(28,11,2,0.9)	8.3	14.99	(30,14)	$c = 8$	15.74	(30,13,7,0.5)	2.1
16.89	(29,14)	$\alpha = 44$	17.73	(29,13,7,0.5)	2.4				16.49	(30,13,3,0.8)	3.8
			18.58	(30,12,4,0.7)	4.1				17.24	(30,13,3,0.8)	4.5
			19.42	(30,12,2,0.9)	5.3				17.78	(30,12,2,0.9)	5.5
			19.56	(30,12,2,0.9)	5.7	15.55	(29,12)	$c = 9$	16.33	(28,12,5,0.6)	3.3
17.29	(31,15)	$\alpha = 46$	18.15	(31,14,6,0.6)	1.8				17.10	(28,11,4,0.7)	5.7
			19.02	(31,14,4,0.8)	2.9				17.78	(28,11,2,0.9)	7.7
			19.88	(31,13,3,0.8)	3.5	16.77	(25,9)	$c = 11$	17.61	(24,8,5,1)	47.1
			20.44	(31,13,2,0.9)	4.3				17.78	(24,8,5,1)	56.4
16.55	(27,12)	$\beta = 2.15$	17.37	(28,11,4,0.7)	4.2	17.42	(21,8)	$c = 12$	17.78	(22,7,0,1)	5.7
			18.20	(27,11,3,0.8)	7.0	15.89	(33,12)	$h = 1$	16.69	(33,11,6,0.5)	2.3
			18.60	(27,11,2,0.9)	8.5				17.48	(33,11,3,0.8)	4.2
16.33	(27,11)	$\beta = 2.20$	17.14	(27,11,5,0.6)	5.0				17.78	(33,11,2,0.9)	4.9
			17.96	(27,10,3,0.8)	9.3	16.01	(30,11)	$h = 1.25$	16.81	(29,11,5,0.6)	3.6
			18.18	(27,10,3,0.8)	10.8				17.61	(29,10,3,0.8)	6.8
15.98	(26,10)	$\beta = 2.30$	16.77	(26,9,6,0.9)	11.8				17.78	(29,10,3,0.8)	7.7
			17.39	(26,9,3,0.8)	19.2	16.29	(24,11)	$h = 1.75$	17.10	(24,10,5,0.6)	15.9
15.80	(25,10)	$\beta = 2.35$	16.59	(25,9,6,1)	15.2				17.78	(24,9,3,0.8)	27.5
			17.02	(25,9,3,0.7)	27.3	16.40	(22,10)	$h = 2$	17.22	(22,9,5,1)	54.3
15.77	(25,9)	$\mu = 3$	16.56	(24,8,4,0.7)	22.6				17.78	(22,9,3,0.8)	86.4
			17.35	(24,8,2,0.9)	37.4	16.14	(27,10)	$b = 20$	16.95	(27,9,5,1)	6.4
			17.78	(24,8,1,0.9)	48.6				17.75	(26,9,2,0.9)	11.3
15.97	(25,10)	$\mu = 4$	16.76	(25,9,4,1)	10.7				17.78	(26,9,2,0.9)	11.6
			17.56	(25,9,2,0.8)	18.1	16.15	(26,11)	$b = 25$	16.96	(26,10,5,0.6)	7.0
			17.78	(25,9,2,0.8)	21.2				17.77	(26,10,2,0.9)	12.1
16.32	(27,12)	$\mu = 6$	17.14	(27,11,6,1)	4.6				17.78	(26,10,2,0.9)	12.2
			17.78	(27,11,4,0.7)	8.4	16.13	(27,11)	$b = 35$	16.94	(26,10,6,1)	7.6
16.52	(28,13)	$\mu = 7$	17.35	(28,12,4,0.8)	3.1				17.74	(26,10,3,0.8)	14.5
			17.78	(28,12,3,0.8)	5.6				17.78	(26,10,3,0.8)	14.9
15.88	(22,12)	$K = 35$	16.68	(22,11,6,0.5)	3.7	16.14	(27,11)	$b = 40$	16.94	(26,11,5,0.6)	8.4
			17.47	(22,11,3,0.8)	6.9				17.75	(26,10,3,0.8)	15.9
			17.78	(22,11,2,0.9)	8.2				17.78	(26,10,3,0.8)	16.3

Average $\Delta\Pi^*$: 13.71 %

4.2 Benefits of temporary price change with a fixed time window T

In Table 2, we observe that in 27 out of 29 problem settings, $T^* \geq 0.8$ for the highest p_2 tested, where $L = 1$. Relying on this observation, we propose a simplified and practical way to solve the model with price change by setting the price change window equal to the lead time ($T = L$). So the problem decision variables reduce to (Q, R, r). We then rerun the problem instances reported in Table 2. The results are reported in Table 3. $\Delta\Pi^T = (\Pi^T(Q^T, R^T, r^T) - \Pi^0(Q^0, R^0))/\Pi^0(Q^0, R^0) * 100$ denotes the percentage improvement provided by a price change optimization model under fixed price change window over a no-price change model.

Setting the price change window equal to the lead time reduces the complexity of the model. It is observed to be a very good approximation. It still generates significant benefits compared to the model with no price change. For the 78 problem instances tested, the average improvement over the no-price change system is 13.10% with $T = L$, down from 13.71% when T was set to be a decision variable. In only 3 out 78 instances, the optimal order quantity Q^T is different from Q^*, though very slightly. In general, for higher values of p_2, r^T is lower than r^*. The result is intuitive, because if the price change is allowed during the whole replenishment lead time, then it can be triggered later during the lead time with a lower trigger-level.

Table 3: Benefit of temporary price change over a fixed price system with a fixed price change window $T = L$. Base setting has $\alpha = 40$, $\beta = 2.25$, $\mu = 5$, $K = 55$, $c = 10$, $L = 1$, $h = 1.5$, and $b = 30$.

p_1	(Q^0, R^0)	Changed Parameter	p_2	(Q^T, R^T, r^T)	$\Delta\Pi^T$ (%)
16.12	(27,11)	-	16.93	(26,10,6)	6.9
			17.74	(26,10,2)	12.0
			17.78	(26,10,2)	12.5
15.81	(25,9)	$\alpha = 38$	16.60	(25,8,5)	29.1
			16.89	(24,8,5)	40.9
16.50	(28,12)	$\alpha = 42$	17.32	(28,11,6)	3.2
			18.15	(28,11,3)	5.1
			18.67	(28,11,2)	7.5
16.89	(29,14)	$\alpha = 44$	17.73	(29,13,5)	1.8
			18.58	(30,12,4)	3.3
			19.42	(30,12,2)	4.9
			19.56	(30,12,2)	5.3
17.29	(31,15)	$\alpha = 46$	18.15	(31,14,6)	1.3
			19.02	(31,14,3)	2.4
			19.88	(31,14,1)	3.2
			20.44	(31,13,1)	4.0
16.55	(27,12)	$\beta = 2.15$	17.37	(27,11,6)	4.0
			18.20	(27,11,2)	6.4
			18.60	(27,11,2)	8.0
16.33	(27,11)	$\beta = 2.20$	17.14	(27,10,5)	4.8
			17.96	(27,10,3)	8.5
			18.18	(27,10,2)	10.2
15.98	(26,10)	$\beta = 2.30$	16.77	(26,9,5)	11.6
			17.39	(26,9,3)	18.7
15.80	(25,10)	$\beta = 2.35$	16.59	(25,9,6)	15.2
			17.02	(25,9,3)	22.5
15.77	(25,9)	$\mu = 3$	16.56	(25,8,3)	21.6
			17.35	(24,8,2)	36.2
			17.78	(24,8,1)	48.1
15.97	(25,10)	$\mu = 4$	16.76	(25,9,4)	10.7
			17.56	(25,9,2)	17.6
			17.78	(25,9,1)	20.5
16.32	(27,12)	$\mu = 6$	17.14	(27,11,6)	4.6
			17.78	(27,11,2)	7.7
16.52	(28,13)	$\mu = 7$	17.35	(28,11,8)	3.0
			17.78	(28,12,3)	4.2
15.88	(22,12)	$K = 35$	16.68	(22,11,5)	3.5
			17.47	(22,11,2)	6.3
			17.78	(22,11,1)	7.7

p_1	(Q^0, R^0)	Changed Parameter	p_2	(Q^T, R^T, r^T)	$\Delta\Pi^T$ (%)
16.01	(25,11)	$K = 45$	16.81	(25,10,6)	3.3
			17.61	(24,11,0)	6.7
			17.78	(24,10,2)	8.2
16.27	(28,11)	$K = 65$	17.09	(28,9,5)	12.3
			17.78	(28,9,3)	20.2
16.38	(30,10)	$K = 75$	17.20	(30,9,5)	20.3
			17.78	(29,9,3)	30.8
14.99	(30,14)	$c = 8$	15.74	(30,13,6)	1.9
			16.49	(30,13,3)	3.1
			17.24	(30,13,2)	4.1
			17.78	(30,13,1)	5.2
15.55	(29,12)	$c = 9$	16.33	(28,12,6)	2.6
			17.10	(28,11,4)	4.8
			17.78	(28,11,2)	7.3
16.77	(25,9)	$c = 11$	17.61	(24,8,5)	47.1
			17.78	(24,8,5)	56.4
17.42	(21,8)	$c = 12$	17.78	(22,7,0)	5.7
15.89	(33,12)	$h = 1$	16.69	(33,11,6)	2.0
			17.48	(33,11,2)	3.8
			17.78	(33,11,1)	4.7
16.01	(30,11)	$h = 1.25$	16.81	(29,11,0)	2.7
			17.61	(29,11,0)	5.5
			17.78	(29,10,2)	6.7
16.29	(24,11)	$h = 1.75$	17.10	(24,9,5)	15.8
			17.78	(24,9,3)	27.0
16.40	(22,10)	$h = 2$	17.22	(22,9,5)	54.3
			17.78	(22,9,3)	84.9
16.14	(27,10)	$b = 20$	16.95	(27,9,5)	6.4
			17.75	(26,9,2)	10.6
			17.78	(26,9,2)	10.9
16.15	(26,11)	$b = 25$	16.96	(26,10,5)	6.5
			17.77	(26,9,3)	11.4
			17.78	(26,9,3)	11.5
16.13	(27,11)	$b = 35$	16.94	(26,10,6)	7.6
			17.74	(26,10,2)	13.3
			17.78	(26,10,2)	13.8
16.14	(27,11)	$b = 40$	16.94	(26,10,6)	8.3
			17.75	(26,10,3)	14.5
			17.78	(26,10,2)	15.0

Average $\Delta\Pi^T$: 13.10 %

4.3 Benefits of temporary price change under a two-stage solution

It is observed in Table 2 that when a price change is allowed, the reorder level is slightly lowered ($R^0 \geq R^*$), but the change in the order quantity can be in either direction ($Q^0 \geq Q^*$ or $Q^0 \leq Q^*$), while all observed changes are mostly negligible. Thus, a simultaneous calculation of replenishment decisions with the price change decisions, which is computationally very challenging, is no longer necessary. Therefore, we also test the sequential evaluation of the price change model by first obtaining (Q^0, R^0) under no price change and then obtaining the best price change trigger-level r^S

with the suboptimal replenishment decisions (Q^0, R^0). As reported in Table 3, setting the price change interval to the lead time $(T = L)$ is a very good approximation of the model with price change that can be easily implemented in practice. Thus, we also set $T = L$ for this sequential optimization solution. In summary, instead of a simultaneous optimization of (Q, R, r) model, two models are solved sequentially: one to obtain the replenishment decisions (Q^0, R^0) and another to obtain the price change decision r^S. The results are reported in Table 4. $\Delta\Pi^S = (\Pi^S(Q^0, R^0, r^S) - \Pi^0(Q^0, R^0))/\Pi^0(Q^0, R^0) * 100$ denotes the percent improvement provided by a sequential price change optimization model under fixed price change window over a no-price change model. Note that p_1 is already reported in Tables 2 and 3, so is not repeated in Table 4.

Table 4: Benefits of two-stage solution of temporary price change model over a fixed price system with $T = L$. Base setting has $\alpha = 40$, $\beta = 2.25$, $\mu = 5$, $K = 55$, $c = 10$, $L = 1$, $h = 1.5$, and $b = 30$.

Changed Parameter	p_2	(Q^0, R^0, r^S)	$\Delta\Pi^S$ (%)
-	16.93	(27,11,0)	3.5
	17.74	(27,11,1)	8.1
	17.78	(27,11,1)	8.3
$\alpha = 38$	16.60	(25,9,2)	21.6
	16.89	(25,9,2)	34.6
$\alpha = 42$	17.32	(28,12,0)	2.2
	18.15	(28,12,1)	4.6
	18.67	(28,12,1)	6.2
$\alpha = 44$	17.73	(29,14,0)	0.7
	18.58	(29,14,0)	1.4
	19.42	(29,14,0)	2.1
	19.56	(29,14,0)	2.2
$\alpha = 46$	18.15	(31,15,1)	0.7
	19.02	(31,15,1)	1.3
	19.88	(31,15,1)	1.9
	20.44	(31,15,1)	2.2
$\beta = 2.15$	17.37	(27,12,2)	2.1
	18.20	(27,12,1)	4.1
	18.60	(27,12,1)	5.1
$\beta = 2.20$	17.14	(27,11,5)	4.4
	17.96	(27,11,1)	7.7
	18.18	(27,11,1)	8.9
$\beta = 2.30$	16.77	(26,10,5)	9.7
	17.39	(26,10,1)	16.6
$\beta = 2.35$	16.59	(25,10,0)	10.2
	17.02	(25,10,0)	16.4

Changed Parameter	p_2	(Q^0, R^0, r^S)	$\Delta\Pi^S$ (%)
$\mu = 3$	16.56	(25,9,2)	17.1
	17.35	(25,9,1)	27.3
	17.78	(25,9,1)	33.7
$\mu = 4$	16.76	(25,10,3)	6.2
	17.56	(25,10,1)	12.7
	17.78	(25,10,1)	14.5
$\mu = 6$	17.14	(27,12,1)	2.5
	17.78	(27,12,1)	5.0
$\mu = 7$	17.35	(28,13,0)	1.7
	17.78	(28,13,0)	2.6
$K = 35$	16.68	(22,12,1)	2.4
	17.47	(22,12,1)	4.6
	17.78	(22,12,1)	5.4
$K = 45$	16.81	(25,11,0)	3.3
	17.61	(25,11,0)	6.6
	17.78	(25,11,1)	7.3
$K = 65$	17.09	(28,11,1)	3.8
	17.78	(28,11,1)	8.5
$K = 75$	17.20	(30,10,1)	11.4
	17.78	(30,10,1)	22.3
$c = 8$	15.74	(30,14,2)	1.2
	16.49	(30,14,2)	2.0
	17.24	(30,14,1)	2.8
	17.78	(30,14,1)	3.3
$c = 9$	16.33	(29,12,5)	2.4
	17.10	(29,12,2)	4.2
	17.78	(29,12,1)	6.1

Changed Parameter	p_2	(Q^0, R^0, r^S)	$\Delta\Pi^S$ (%)
$c = 11$	17.61	(25,9,1)	30.4
	17.78	(25,9,1)	40.7
$c = 12$	17.78	(21,8,0)	3.2
$h = 1$	16.69	(33,12,1)	1.5
	17.48	(33,12,1)	2.7
	17.78	(33,12,1)	3.2
$h = 1.25$	16.81	(30,11,0)	2.7
	17.61	(30,11,0)	5.4
	17.78	(30,11,1)	6.0
$h = 1.75$	17.10	(24,11,1)	5.3
	17.78	(24,11,1)	11.9
$h = 2$	17.22	(22,10,1)	31.6
	17.78	(22,10,1)	63.3
$b = 20$	16.95	(27,10,5)	5.2
	17.75	(27,10,1)	9.4
	17.78	(27,10,1)	9.5
$b = 25$	16.96	(26,11,0)	2.7
	17.77	(26,11,1)	6.4
	17.78	(26,11,1)	6.4
$b = 35$	16.94	(27,11,2)	4.6
	17.74	(27,11,1)	9.9
	17.78	(27,11,1)	10.2
$b = 40$	16.94	(27,11,3)	5.9
	17.75	(27,11,2)	12.0
	17.78	(27,11,2)	12.2

Average $\Delta\Pi^S$: 9.10 %

When the replenishment decisions are obtained sequentially, and the price change is allowed, we observe that $r^S \leq r^*$. The rationale is to correct for the effects of suboptimal replenishment decisions and the price change window. The price change trigger-level is lowered postponing a change in price. However, the resulting expected profits are still good enough compared to a fixed price policy. The average profit gain is 9.1% for the tested instances in Table 4. Thus, even with a sequentially solved replenishment and price change problem, the resulting decisions provide significant improvements over a conventional single price policy.

4.4 Results from randomly generated problems

Next, we generate 100 additional problem instances to repeat the above analyses. Each parameter is randomly generated for each problem instance from Uniform distributions with ranges listed in Table 5, with L set to 1. The range of distribution is wide enough to test the robustness of the results. Note that like in the above analyses, for each problem setting, the system performance is tested by setting price p_2 to various values between $1.05p_1$ and α/β with multiples of 5% increments on p_1. A total of 309 instances with a price change are tested, corresponding to 100 problem instances with no price change.

Table 5: Distributions of Randomly Generated Parameters.

α	U(38, 46)	μ	U(3, 7)	K	U(35, 75)	c	U(8, 12)
β	U(2.1, 2.4)	h	U(1, 2)	b	U(5, 50)		

For a total of 309 problems instances with a price change, our results demonstrate an average profit improvement of $\Delta\Pi^*$=8.11% provided by the price change policy that is optimized over variables (Q, R, r, T). For these tested problems, the standard deviation of $\Delta\Pi^*$ is 13.78, the minimum being 0.79% and the maximum 97.68%. When the price change window T is set to L, optimization of (Q, R, r) model results in an average profit improvement of $\Delta\Pi^T$=7.64% over the single price policy with the standard deviation 13.71%, minimum 0.33%, and maximum 96.32%. Moreover, the performance of sequential optimization of price change policy is also tested with these randomly generated problems. When the price change trigger-level r is optimized, the resulting policy provides an average increase of $\Delta\Pi^S$=5.15% in expected profits with a standard deviation of 10.2%, minimum of 0.21%, and maximum of 81.25%.

In summary, the numerical analyses indicate a high potential to improve the expected profits by making only a single price change during the replenishment lead time. Although it is theoretically optimal to make all of the inventory replenishment and price decisions simultaneously, the proposed simplified policies demonstrate considerable improvements over the conventional single price policy. First, the price change decision can be limited to a trigger inventory level without being constrained to a time window, resulting in a three decision variable problem (Q, R, r), easier to implement in practice. Second, this three-decision problem can be solved sequentially, by first obtaining the replenishment policy (Q, R) and then solving for the trigger inventory level r. Such a sequential solution is simple and easy to compute, and results in significant profit improvement. In a sequential optimization, the optimal price change trigger-level is lower than the one in simultaneous optimization. Even with a low trigger-level, the expected profit can still be improved, due to the better use of the remaining inventory and ability to manage demand to avoid shortages.

5. Concluding Remarks

In this study, we consider a continuous review inventory model with a fixed cost and non-zero lead time, and introduce a temporary price increase exercised over a particular time window during the lead time. This business practice is common among retailers, such as Amazon, Walmart, and Apple. It helps in better managing demand and increases revenues. This problem has not been addressed before in the dynamic pricing literature. Furthermore, none of the work on dynamic pricing considered a continuous review model with a fixed cost has assumed a non-zero lead time. We solve for the continuous review inventory replenishment and temporary pricing decisions. The price change policy is defined such that if the inventory level in a time window within the replenishment lead time drops to a critical level, the regular market price is increased. The corresponding trigger inventory level and time window are two decision variables that define the price change policy. The resulting optimization problem includes two replenishment decisions and two price change decisions.

We develop the long-run expected profit function. As the analytical tractability of the problem is highly challenging, we conduct extensive numerical and sensitivity analyses to analyze the temporary price change policy. The results indicate that a model with joint replenishment and price change decisions provides significant improvements in expected profits (13% on average) over a conventional single price model. We further simplify the proposed model by allowing price changes to occur over the entire lead time period, and still observe significant improvements in profit. We then propose a sequential optimization approach to solve for the decision variables as a heuristic to the original model. Numerical analyses demonstrate that proposed two-stage optimization approach results in considerable profit improvements (9% on average), thus it can be easily implemented in practice. More complex systems including price optimization with more than on price change can be built on the premise of improvements provided by this simple policy.

Our results indicate that a temporary price increase is mostly beneficial when the unit purchasing cost, fixed order cost, unit holding cost, or the unit lost sale cost is high. The price increase policy yields a better use of the available inventory when the optimal reorder level and order sizes are low with the increased costs. Moreover, exercising a temporary price increase is more attractive when the demand rate is low such as for slow-moving items and also for seasonal products experiencing a surge in demand.

This study is the first research in the Operations literature to investigate particularly temporary price increase policies. The time to change the price could depend on the on-hand inventory and the time until receiving the next order. Besides, a firm could exercise multiple price changes during the lead time. However, each additional decision would complicate the model and its practical applicability. Thus, we could extend this work in light of these observations.

References

Archibald, B. C. 1981. Continuous Review (s, S) Policies with Lost Sales. *Management Science* **27**(10) p.1171-1177.

Baker, W., M. V. Marn, C. Zawada. 2001. Price Smarter on the Net. *Harvard Business Review* **79** p.122-127.

Bhargava, H.K., D. Sun, S.H. Xu. 2006. Stockout Compensation: Joint Inventory and Price Optimization in Electronic Retailing. *INFORMS Journal on Computing* 18(2) p.255-266.

Boudette, N.E. 2014. Quake Throws AutoNation Into a Spin. *The Wall Street Journal*. *http : //online.wsj.com/news/ articles/SB*10001424052748704281504576324972233601388. Retrieved on July 30, 2014.

Businesswire.com. 2014. Upstream commerce research reveals Amazon regularly raises prices for cyber monday.
*https : //www.businesswire.com/news/home/*20141112006615*/en/Upstream − Commerce − Research − Reveals − Amazon − Regularly − Raises*, Retrieved on August 1, 2018.

Chan W. 2015. 4 Reasons why iPhone LCD screen Price are Rising. Linkedin.com

Chan, L. M. A., Z. J. Max Chen, D. Simchi-Levi, J. L. Swann. 2004. Coordination of Pricing and Inventory Decisions: A Survey and Classification, in *Handbook of Quantitative Supply Chain Analysis: Modeling in the E-Business Era*, D. Simchi-Levi, S.D. Wu and Z.J.M. Shen (eds.), Kluwer Academic Publishers, p.335-392.

Chao, X., S. X. Zhou 2006. Joint Inventory-and-Pricing Strategy for a Stochastic Continuous-Review System. *IIE Transactions* 38 p.401-408.

Chen, F. Y., S. Ray, Y. Song. 2006. Optimal Pricing and Inventory Control Policy in Periodic-review Systems with Fixed Ordering Cost and Lost Sales. *Naval Research Logistics* 53(2) p.117-136.

Chen, H., O. Q. Wu, D. D. Yao. 2010. On the Benefits of Inventory-Based Dynamic Pricing Strategies. *Production and Operations Management* 19(3) p.249-260.

Chen, X., D. Simchi-Levi. 2004. Coordinating Inventory Control and Pricing Strategies with Random Demand and Fixed Ordering Cost: The Infinite Horizon Case. *Mathematics of Operations Research* 29(3) p.698-723.

Chen, X., D. Simchi-Levi. 2006. Coordinating Inventory Control and Pricing Strategies: The Continuous Review Model. *Operations Research Letters* 34(3) p.323-332.

Cheung, K. L. 1998. A Continuous Review Inventory Model with a Time Discount. *IIE Transactions* 30(8) p.747-757.

Clemons, E. K., B.W. Webber. 1997. Information technology and screen-based securities trading: pricing the stock and pricing the trade. *Management Science*, 43(12) p. 1693-1708.

Corsten, D., T. Gruen. 2004. Stock-Outs Cause Walkouts. *Harvard Business Review* 82(5) p.26-28.

Çömez, N., T. Kiessling. 2012. Joint Inventory and Constant Price Decisions for a Continuous Review System. *International Journal of Physical Distribution & Logistics Management*, 42(2),

p. 174-202.

Çömez, N., K. E. Stecke, M. Çakanyıldırım. 2012. Multiple In-Cycle Transshipments with Positive Delivery Times. *Production and Operations Management* 21(2) p.378-395.

DeCroix, G.A., A. Arreola-Risa. 1998. On Ordering Economic Incentives to Backorder. *IIE Transactions* 30(8) p.715-721.

Dewan, R. M., M.L. Freimer, Y. Jiang. 2007. A temporary monopolist: taking advantage of information transparency on the web.*Journal of Management Information Systems*, 24(2) p.167-194.

Ding, Q., P. Kouvelis, J.M. Milner. 2007. Dynamic Pricing for Multiple Class Deterministic Demand Fulfillment. *IIE Transactions* 39(11) p.997-1013.

Drake, M.J., D.W. Pentico. 2011. Price Discounts for Increased Profitability Under Partial Backordering. *International Transactions in Operational Research* 18(1) 87-101.

Durán, A., G. Gutiérrez, R. I. Zequeira. 2004. A Continuous Review Inventory Model with Order Expediting. *International Journal of Production Economics* 87 p.157-169.

Elmaghraby, W. P. Keskinocak. 2003. Dynamic Pricing in the Presence of Inventory Considerations: Research Overview, Current Practices, and Future Directions. *Management Science* 49(10) p.1287-1309.

Federgruen, A., A. Heching. 1999. Combined Pricing and Inventory Control Under Uncertainty. *Operations Research* 47 p.454-475.

Feng, Y., G. Gallego. 1995. Optimal Starting Times for End-of-Season Sales and Optimal Stopping Times for Promotional Fares. *Management Science* 41(8) p.1371-1391.

Fersht, P. 2013. Why American firms are more progressive with outsourcing than the Europeans and Asians. $https://www.horsesforsources.com/us_eu_apac_041713$. Retrieved on August 1, 2018.

Gallego, G., G. J. van Ryzin. 1994. Optimal Dynamic Pricing of Inventories with Stochastic Demand Over Finite Horizons. *Management Science* 40 p.999-1020.

Gayon, J.P., I. Talay-Degirmenci, F. Karaesmen, E.L. Örmeci. 2009. Optimal Pricing and Production Policies of a Make-to-Stock System with Fluctuating Demand. *Probability in the Engineering and Informational Sciences* 23(2) p.205-230.

Gimpl-Heersink, L., C. Rudloff, M. Fleischmann, A. Taudes. 2008. Integrating Pricing and Inventory Control: Is it Worth the Effort? *Business Research* 1(1) p.106-123.

Gracey, K. 2014. RFID Tag Prices Dropped 70%. http://www.parallax.com/news/2014-07-14/rfid-tag-prices-dropped-70-and-unfortunate-reason-they-were-so-high-last-year. Retrieved on July 30, 2014.

Hadley, G., T. Whitin. 1963. *Analysis of Inventory Systems*, Prentice-Hall, Englewood Cliffs, N.J.

Jadidi, O., M.Y. Jaber, S. Zolfaghari. 2017. Joint pricing and inventory problem with price dependent stochastic demand and price discounts. *Computers and Industrial Engineering* 114, p.45-53.

Johansen, S. G., A. Thorstenson. 1998. An Inventory Model with Poisson Demands and Emergency Orders. *International Journal of Production Economics* 56-57 p.275-289.

Lee, S.Y. 2017. As iPhone looms, firms scramble to lock up memory chip supply. *Reuters*. *https : //www.reuters.com/article/us − tech − chips − analysis/as − iphone −* 8 *− looms − firms − scramble − to − lock − up − memory − chip − supply − idUSKBN*19*B*39*S*. Retrieved on August 1, 2018.

Lau, A.H.-L., H.-S. Lau. 2002. A Comparison of Different Methods for Estimating the Average Inventory Level in a (Q, R) System with Backorders. *International Journal of Production Economics* 79(3) p.303-316.

Long, M. 2017. SSDs are about to skyrocket in cost: Should you upgrade in 2017? *Makeuseof.com*. https://www.makeuseof.com/tag/ssds-skyrocket-cost-upgrade, Retrieved on August 1, 2018.

Moinzadeh, K., S. Nahmias. 1988. A Continuous Review Model for an Inventory System with Two Supply Modes. *Management Science* 34 p.761-773.

Mochizuki, T. 2017. Nintendo battles Apple for parts as Switch demand rises. The Wall Street Journal. *https : //www.wsj.com/articles/nintendo − battles − apple − for − parts − as − switch − demand − rises −* 1496136603, Retrieved on August 1, 2018.

Petruzzi, N. C., M. Dada. 1999. Pricing and the Newsvendor Problem: A Review with Extensions. *Operations Research* 47(2) p.183-194.

Pmcofiowa.com. 2005. The Factors Influencing the Price of Gasoline. http://www.pmcofiowa.com.

Quint, M., D. Shorten. 2005. The China Syndrome. *strategy+business*. *https : //www.strategy − business.com/article/*05102?*gko* = 3875*c*, Retrieved on August 1, 2018.

Ray, S., S. Li, Y. Song. 2005. Tailored Supply Chain Decision Making Under Price-Sensitive Stochastic Demand and Delivery Uncertainty. *Management Science* 51(12) p.1873-1891.

Retailwire.com. 2014. Amazon raises prices on Cyber Monday. *https : //www.retailwire.com/discussion/amazon raises − prices − on − cyber − monday/*, Retrieved on August 1, 2018.

Sellics.com. 2016. Going out of stock vs increasing prices: what to do when running low on inventory on Amazon(Case Study). *https : //sellics.com/blog − out − of − stock − vs − price − increase − amazon − inventory*, Retrieved on August 1, 2018.

Serel, D. 2009, Optimal Ordering and Pricing in a Quick Response System. *International Journal of Production Economics* 121 p.700-714.

Taylor, A. 2006. Top toys command eBay premium.*The Financial Times*. *https : //www.ft.com/content/f*98*e*6684 9051 − 11*db* − *a*4*b*9 − 0000779*e*2340. Retrieved on July 29, 2018.

Tekin, E., U. Gurler, E. Berk. 2001. Age-based vs. Stock Level Control Policies for a Perishable Inventory System. *European Journal of Operational Research* 134 p.309-329.

Thomas, D.J., J.E. Tyworth. 2006. Pooling Lead-time Risk by Order Splitting: A Critical Review. *Transportation Research Part E: Logistics and Transportation Review* 42(4) p.245-257.

Tomlin, B. 2006. On the Value of Mitigation and Contingency Strategies for Managing Supply Chain Disruption Risks. *Management Science* 52(5) p.639-657.

Vitasek, K.L., K.B. Manrodt, M. Kelly. 2003. Solving the Supply-demand Mismatch. *Supply Chain Management Review* September October 58-64.

Whitin, T. M. 1955. Inventory Control and Price Theory. *Management Science* 2 p.61-68.

Xu, Y., A. Bisi, M. Dada. 2011. A Periodic-Review Base-Stock Inventory System with Sales Rejection. *Operations Research* 59(3) p.742-753.

Yano, C. A., S. M. Gilbert. 2005. Coordinated Pricing and Production/Procurement Decisions: A Review *Managing Business Interfaces*, A. K. Chakravarty, J. Eliashberg (eds.), Springer U.S., p.65-103.