Integrated manufacturer-retailer closed-loop inventory system with price-sensitive return and demand rates

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Abstract

This study developed an integrated manufacturer-retailer closed-loop inventory system with price-sensitive return and demand rates. Glass bottles and printer cartridges are examples of products that can be recycled and used as new components. The demand rate is price-sensitive to the selling price, and the return rate depends on the return price of a recycled component. This paper considered four scenarios with different strategies and solution procedures. The purpose of this study was to optimize the product selling price, component return price, the lot sizes of new and returned components, and the lot size of finished products delivered to the retailer. A numerical example was provided to illustrate the proposed model. Sensitivity analyses on the return price, selling price, purchase cost, and other parameters were conducted. In addition, a comparison on the differences between considering and not considering recycling and/or integration was presented.

***Keywords**:* Inventory; Recycled component; Closed-loop system; Price-sensitive demand rate; Price-sensitive return rate

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1. Literature Review

Manufacturer-retailer system-wide integration is essential to successful supply chain management. Integration can occur for problems such as supplier selection, product recovery, and inventory planning, etc. Clark and Scarf (1960) presented the concept of serial multi-echelon structures to determine the optimal policy. Banerjee (1986) developed a joint economic lot-size model for a single-vendor–single-buyer system in which the vendor has a finite production rate. Goyal (1988) relaxed the lot-for-lot assumption for the model by Banerjee.

Von Stackelberg (1952) introduced a technique to solve the two-phase decision-making problem within a hierarchical decision making when both the manufacturer and the retailer have no direct control upon the decision of the other but their actions affect the subsequent responses from the other. Liou et al. (2006) considered the Stackelberg equilibrium framework in which the economic order quantity is integrated with the economic production quantity. The objective is to maximize the vendor’s total benefit subject to the minimum total cost that the buyer is willing to incur. Yu et al. (2009) modeled a Stackelberg game where one manufacturer is the leader and multiple retailers follow their own optimal product pricing and inventory policies.

More recently, Mendoza and Ventura (2012) developed models for integrated supplier selection and order quantity allocation with consideration of quality and capacity factors for the suppliers under the assumption that suppliers are always available. Özceylan et al. (2014) proposed an integrated model that jointly optimizes strategic level decisions related to the amounts of goods flowing on the forward and reverse chains and tactical level decisions concerning balancing the disassembly lines in the reverse chain. Pazhani et al. (2015) considered a serial inventory system integrating inventory management with supplier selection decisions and transportation costs. In their analysis, Pazhani et al. (2015) discussed the benefits of integration by comparing their system to a sequential system. Mota et al. (2018) studied sustainable supply chains and found the importance of having an integrated approach that integrates decisions at various levels of the supply chain.

Koh et al. (2002) developed an optimal new-product ordering (EOQ, Economic Ordering Quantity) and recycled-product recovery (EPQ, Economic Production Quantity) policy for reusable items with a stationary demand. Guide et al. (2003) investigated profit maximization from remanufacturing in which the profitability depends on the price-sensitive demand for remanufactured products, as well as on the quantity and quality of the product returns that can be influenced by varying quality-dependent acquisition prices. Teunter (2004) derived a simple formula that determines the optimal lot sizes for the production/procurement of new products and for the recovery of returned products. Choi et al. (2007) generalized Koh et al.'s model by treating the sequence of orders for newly purchased products and setups for the recovery process within a cycle as a decision variable. Rubio and Corominas (2008) presented an optimal manufacturing and remanufacturing policy in which the manufacturing, remanufacturing and return rates are decision variables. Jaber and Saadany (2009) assumed a CLSC with allowable shortage when the demand for manufactured items is different from that for remanufactured ones. Feng (2011) developed a generalized (P, R) policy in which there are P manufacturing setups followed by R remanufacturing setups, instead of (1, R) and (P, 1) policies. The demands of the above-mentioned researches were assumed to be constant.

Shi et al. (2011) addressed an optimal production and pricing policy when the demand is uncertain and sensitive to the selling price and the return rate is dependent on the acquisition of the used products. Kabirian (2012) developed an economic production and pricing model with various patterns of price-sensitive demand. Georgiadis and Athanasiou (2013) developed demand-driven capacity planning policies for uncertain demand. The above-mentioned researches focus on the discussion of finished product recycling. Wang et al. (2016) proposed a remanufacturing/manufacturing system where returns are collected under a name-your-own-price (NYOP) bidding mechanism, which was then compared with the traditional list-price mechanism. The result showed that manufacturers prefer the NYOP mechanism under the conditions of a low reverse market share, a high manufacturing cost, sufficient capacity, or a low capacity requirement of remanufacturing. Hong et al. (2016) studied a four-tiered network equilibrium model consisting of the sources of electronic scrap products and the collectors, processors and demand markets. The study investigated the impacts of exogenous government subsidies on recycled material flows in decentralized reverse supply chains.

This study focused on component recycling instead of product recycling to optimize ordering and recycle policies for an integrated manufacturer-retailer CLSC. The demand was assumed to be price-sensitive and the return rate was dependent on the return price of the recycled component. A numerical example was provided to illustrate the mathematical model. Sensitivity analysis on the return price, selling price, and other parameters were conducted, and a comparison on the differences between considering and not considering the return policy was presented.

2. Model Development

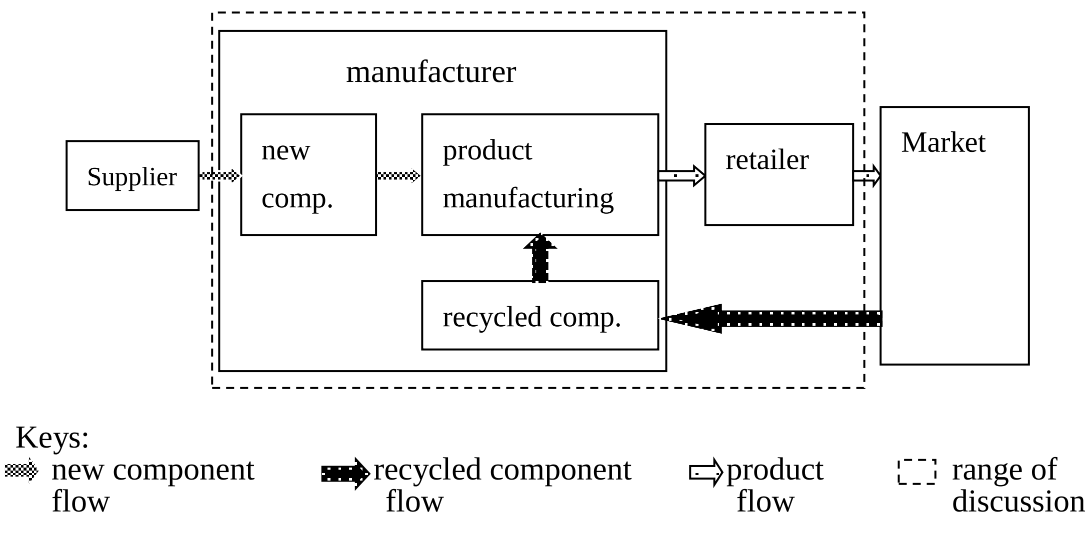

Figure 1. Component and product flow in a closed-loop system

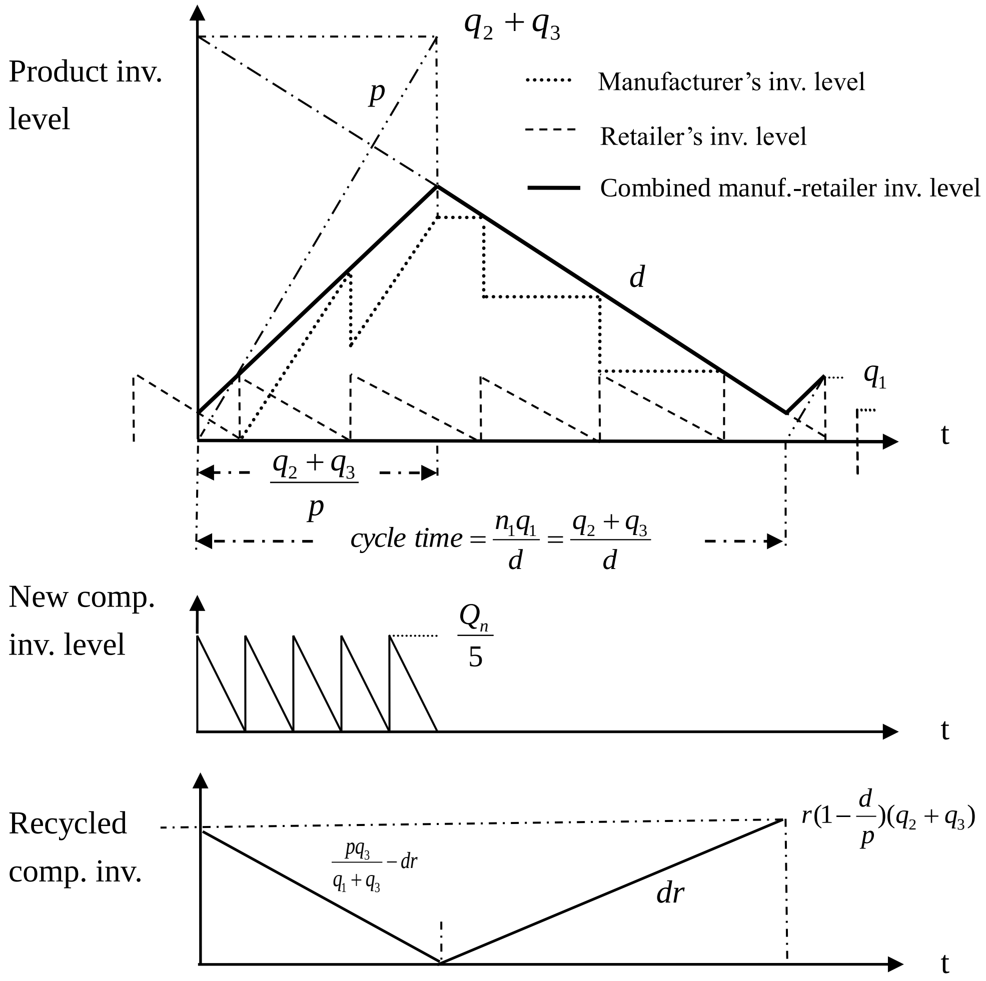

Figure 2. Inventory levels of finished products, new, and recycled components for n_1=5, n_2=5

The mathematical model was based on the following assumptions:

(a) A single manufacturer and a single retailer.

(b) The recycled component is comparable to the new component.

(c) The system contains a single item in an infinite planning horizon.

(d) The demand rate is linearly price-sensitive and lower than the finite production rate.

(e) The component's return rate is linearly price-sensitive with the return price, and the value of return rate is between 0 and 1.

(f) The manufacturer and retailer's unit holding costs are constant, and the cost of the latter is higher than that of the former.

(g) The unit holding cost of the return is constant.

(h) The replenishment intervals of the new component and the finished product are constant.

(i) The component unit usage for each finished product is one.

Independent decision variables:

q_1: Lot size received by the retailer

ρ: Selling price to the end consumer

n_1: Number of product deliveries to the retailer in each manufacturing cycle (positive integer)

n_2: Number of deliveries of new components to manufacturers during the production period in each manufacturing cycle (positive integer)

u: Unit return price of the recycled components, including inspection, repairs, and other related costs

Retailer parameters:

d: Annual demand rate, where $d = \alpha_1 - \beta_1 \rho$; in which α_1 is the demand scale parameter and β_1 is the price-sensitive parameter

H_r: Annual holding cost per unit

O_r: Ordering cost per time

Manufacturer product parameters:

w: Wholesale price

p: Annual production rate

H_1: Annual holding cost per unit

s: Setup cost per period

Manufacturer new component parameters:

m: Unit purchase cost

H_2: Annual holding per unit

O_2: Ordering cost per time

q_2: Total quantity during the production period in each manufacturing cycle (dependent variable)

Manufacturer recycled component parameters:

r: Percentage return (or recycling) based on demand, where $r = \alpha_2 + \beta_2 u$; in which α_2 is the return scale parameter, β_2 is the return price-sensitive parameter, and $0 \le r \le 1$

q_3: Total quantity during the production period in each manufacturing cycle (dependent variable)

H_3: Annual holding cost per unit

Other related notations:

TP_r: Total profit of the retailer

TP_m: Total profit of the manufacturer

TP: Total profits of the retailer and manufacturer

The new, recycled components, and product flow are shown in the closed-loop manufacturer-retailer system as illustrated in Figure 1. The manufacturer orders new components from the supplier with Just-in-time (JIT) delivery, and the recycled components are returned (recycled) from the market to the manufacturer's warehouse as new components. The manufacturer delivers products to the retailer as soon as there is enough to make up the batch size, thus reducing the inventory cost during the production period. The new components and the recycled components are conveyed to the production line simultaneously. The inventory levels for finished products, as well as new and recycled components, are depicted in Figure 2. The first diagram includes the product inventory levels for the manufacturer, the retailer and the combined manufacturer-retailer. The cycle time is the sum of the total quantities of new and recycled components ($q_2 + q_3$) divided by the demand rate (d), that is $(q_2 + q_3)/d$. In each cycle, the length of the production time is $(q_2 + q_3)/p$, and the length of the non-production time is $(q_2 + q_3)(1/d - 1/p)$. The number of product deliveries to the retailer in each manufacturing cycle is five (i.e., n_1=5). The second diagram is a new component inventory level. During production, the supplier delivers new components in five lots (i.e., n_2=5), and each lot size is $(q_2/5)$. The third diagram is the recycled component inventory level. During non-production, the recycled component inventory level is the annual increased recycled quantity multiplied by the length of the non-production time; that is $(dr)(q_2+q_3)(1/d-1/p)=r(1-d/p)(q_2+q_3)$. Since the total quantities of the new and recycled components during each production time are q_2 and q_3 respectively, the production rate using recycled components is $pq_3/(q_2+q_3)$. The slope of the recycled component inventory level is $pq_3/(q_2+q_3)-dr$.

The annual total profit of the retailer is expressed as:

$$TP_r = \rho d - wd - \frac{q_1}{2}H_r - \frac{d}{q_1}O_r, \quad where\ d = \alpha_1 - \beta_1\rho. \tag{1}$$

The first term on the right side in (1) is the sales revenue, the second term is the purchase cost, and the third and fourth terms are the holding and ordering costs. The duration of the manufacturing cycle is $\frac{q_2+q_3}{d}$, and the annual average inventory level is

$$\frac{q_2}{2n_2}\frac{q_2+q_3}{p}/(\frac{q_2+q_3}{d}) = \frac{q_2 d}{2n_2 p}.$$

The manufacturer's new component-related cost per year is:

$$\frac{q_2 d}{2n_2 p}H_2 + \frac{d}{q_2}O_2 n_2 + md\frac{q_2}{q_2+q_3} \tag{2}$$

The first, second, and third terms in (2) are the holding, ordering, and purchase costs, respectively.

The manufacturer's return component-related cost is:

$$\frac{1}{2}(r(1-\frac{d}{p})(q_2+q_3))H_3 + udr \tag{3}$$

The first and second terms in (3) are the holding and purchase costs, respectively.

During each manufacturing cycle, as shown in Figure 1, the production quantity is n_1q_1, the delivery lot size to the retailer is q_1, and the manufacturer's average product inventory level is $\frac{q_1}{2}((n_1-1)(1-\frac{d}{p})+\frac{d}{p})$ (see Yang and Wee, 2007).

The manufacturer's product-related cost is:

$$\frac{q_1}{2}((n_1-1)(1-\frac{d}{p})+\frac{d}{p})H_1+\frac{d}{q_2+q_3}s \tag{4}$$

The first and second terms in (4) are the holding and setup costs, respectively. The total profit of the manufacturer is the sales revenue minus the related costs of (2), (3), and (4):

$$\begin{aligned} TP_m = dw-(\frac{q_2dH_2}{2n_2p}+\frac{d}{q_2}O_2n_2+md\frac{q_2}{q_2+q_3})-\{\frac{q_1}{2}((n_1-1)(1-\frac{d}{p})+\frac{d}{p})H_1 \\ +\frac{d}{q_2+q_3}s\}-\{\frac{1}{2}(r(1-\frac{d}{p})(q_2+q_3))H_1+udr\} \end{aligned} \tag{5}$$

4. The Four Scenarios and Solution Procedures

For the purpose of investigating the importance of recycling and integration, four scenarios were discussed: (1) recycling and integration; (2) recycling without integration; (3) without recycling but with integration; and (4) without recycling or integration.

Scenario 1: Recycling and integration

The objective is to maximize the system profits of the retailer and the manufacturer:

$$\begin{aligned} Max\, TP &= TP_m + TP_r \\ &= \rho d-\frac{q_1}{2}H_r-\frac{d}{q_1}O_r-\{\frac{1}{2}[r(1-\frac{d}{p})(q_2+q_3)]H_3+udr\} \\ &\quad -\{\frac{q_1}{2}[(n_1-1)(1-\frac{d}{p})+\frac{d}{p}]H_1+\frac{d}{q_2+q_3}s\} \\ &\quad -[\frac{d}{q_2}O_2n_2+\frac{q_2dH_2}{2n_2p}+md\frac{q_2}{q_2+q_3}] \end{aligned} \tag{6}$$

Subject to:

$$d = \alpha_1 - \beta_1 \rho, \quad d > 0 \tag{7}$$
$$r = \alpha_2 + \beta_2 u, \quad 0 \le r \le 1, \quad u \ge 0 \tag{8}$$
$$q_3 = n_1 q_1 r \tag{9}$$

and

$$q_2 = n_1 q_1 (1 - r) \tag{10}$$

The linearly price-sensitive demand is constrained in (7), where $\alpha_1 \ge 0, \beta_1 > 0, d > 0$. The return rate is dependent on the return price of the recycled component, as illustrated in (8), where $\alpha_2 \ge 0, \beta_2 > 0$. Constraints (9) and (10) show the following two relationships. First, during each cycle, the sum of the new and recycled components is equal to the lot size q_1 multiplied by delivery periods N_s; that is, $q_2 + q_3 = n_1 q_1$. Second, the lot size ratio of the recycled and the new components is $q_3 : q_2 = r : (1 - r)$. Substituting (7) through (10) into (6) yields five independent decision variables: n_2, n_1, u, ρ and q_1.

The solution procedures are as follows:

(i) After substituting (7), (8), (9), and (10) into (6), solve the following equations simultaneously: $\{\frac{\partial}{\partial \rho} TP = 0, \quad \frac{\partial}{\partial n_1} TP = 0, \quad \frac{\partial}{\partial n_2} TP = 0, \quad \frac{\partial}{\partial q_1} TP = 0, \quad \frac{\partial}{\partial u} TP = 0\}$.

(ii) Let the values of N_n and N_s be positive integers in the neighborhood of the values from (i) and solve the following equations simultaneously: $\{ \frac{d}{d\rho} TP = 0, \frac{d}{dq_1} TP = 0, \frac{d}{du} TP = 0 \}$. Then, compute the total profit in (6).

(iii) The optimal value of $\{ n_2$ and $n_1 \}$ must satisfy the following conditions:

$$TP(n_2 - 1, n_1, \rho, q_1, u) \le TP(n_2, n_1, \rho, q_1, u) \ge TP(n_2 + 1, n_1, \rho, q_1, u) \tag{11}$$

and

$$TP(n_2, n_1 - 1, \rho, q_1, u) \le TP(n_2, n_1, \rho, q_1, u) \ge TP(n_2, n_1 + 1, \rho, q_1, u) \tag{12}$$

Scenario 2: Recycling without integration

The objective is to maximize a two-phase objective function:

$$Phase\ 1: Max\ TP_r\ in\ (1) \tag{13}$$
$$Subject\ to\ (7)$$
$$Phase\ 2: Max\ TP_m\ in\ (5) \tag{14}$$
$$Subject\ to\ (7), (8), (9)\ and\ (10)$$

Two independent decision variables, $\{\rho, q_1\}$, in (13), and three independent decision variables, $\{n_2, n_1, u\}$, in (14) are considered.

The solution procedures are as follows:

(i) Phase 1: Determine the optimal total profit of the retailer in (13).

Substituting $d = \alpha_1 - \beta_1\rho$ into (1), solve the following simultaneous equations: $\{\frac{\partial TP_r}{\partial \rho} = 0, \frac{\partial TP_r}{\partial q_1} = 0\}$.

(ii) Phase 2: Determine the optimization of the manufacturer's total profit in (14).

Substituting ρ, q_1, (7), (8), (9), and (10) into (14) yields three decision variables, $\{n_2, n_1, u\}$. Solve the simultaneous equations $\{\frac{\partial}{\partial n_1} TP_m = 0, \frac{\partial}{\partial n_2} TP_m = 0, \frac{\partial}{\partial u} TP_m = 0\}$.

(iii) Let the values of n_2 and n_1 be positive integers in the neighborhood of the values from (ii). Solve the equation $\{\frac{d}{du} TP_m = 0\}$. Then, compute the manufacturer's total profit in (14).

(iv) The optimal value of $\{n_2 \text{ and } n_1\}$ must satisfy the following conditions:

$$TP_m(n_2 - 1, n_1, \rho, q_1, u) \le TP_m(n_2, n_1, \rho, q_1, u) \ge TP_m(n_2 + 1, n_1, \rho, q_1, u) \tag{15}$$

and

$$TP_m(n_2, n_1 - 1, \rho, q_1, u) \le TP_m(n_2, n_1, \rho, q_1, u) \ge TP_m(n_2, n_1 + 1, \rho, q_1, u) \tag{16}$$

Scenario 3: Without recycling but with integration

Without considering recycling, let $r = 0, u = 0, q_3 = 0$. The objective is to maximize the system profits of the retailer and the manufacturer:

$$TP = TP_m + TP_r$$
$$= \rho d - \frac{q_1}{2}H_r - \frac{d}{q_1}O_r - \{\frac{q_1}{2}[(n_1-1)(1-\frac{d}{p}) + \frac{d}{p}]H_1 + \frac{d}{q_2}s\}$$
$$-[\frac{d}{q_2}O_2 n_2 + \frac{q_2 dH_2}{2n_2 p} + md] \tag{17}$$

Subject to (7) and $q_2 = n_1 q_1$.

The solution procedures are as follows:

(i) Substituting (7) and $q_2 = n_1 q_1$ into (17) yields four independent decision variables $\{n_2, n_1, \rho, q_1\}$. Take the first derivatives of (17) with respect to n_2, n_1, ρ, and q_1 to zero. Solve the following simultaneous equations: $\{\frac{\partial}{\partial \rho}TP = 0, \frac{\partial}{\partial n_1}TP = 0, \frac{\partial}{\partial n_2}TP = 0, \frac{\partial}{\partial q_1}TP = 0\}$.

(ii) Let the values of n_2 and n_1 be positive integers in the neighborhood of the values from (i) and solve the following simultaneous equations: $\{\frac{d}{d\rho}TP = 0, \frac{d}{dq_1}TP = 0\}$. Then, compute the total profit in (17).

(iii) The optimal value of $\{n_2$ and $n_1\}$ must satisfy the following conditions:

$$TP(n_2 - 1, n_1, \rho, q_1) \le TP(n_2, n_1, \rho, q_1) \ge TP(n_2 + 1, n_1, \rho, q_1) \tag{18}$$

and

$$TP(n_2, n_1 - 1, \rho, q_1) \le TP(n_2, n_1, \rho, q_1) \ge TP(n_2, n_1 + 1, \rho, q_1) \tag{19}$$

Scenario 4: Without recycling or integration

Let $r = 0, u = 0, q_3 = 0$. The objective is to maximize a two-phase function:

$$Phase\,1: Max\, TP_r = (\rho - w)d - \frac{q_1}{2}H_r - \frac{d}{q_1}O_r$$
$$Subject\ to\ (7) \tag{20}$$

After substituting (7) into (20), there are two independent decision variables, $\{\rho, q_1\}$ in (20).

$$Phase\ 2: Max\ TP_m = dw - [\frac{d}{q_2}O_2 n_2 + \frac{q_2 d H_2}{2n_2 p} + md] - \{\frac{q_1}{2}[(n_1-1)(1-\frac{d}{p}) + \frac{d}{p}]H_1 + \frac{d}{q_2}s\} \quad (21)$$
$$Subject\ to\ (7)\ and\ q_2 = n_1 q_1.$$

The solution procedures are as follows:

(i) Phase 1: Determine the optimal retailer's profit in (20). Solve the following simultaneous equations: $\{ \frac{\partial TP_r}{\partial \rho} = 0, \frac{\partial TP_r}{\partial q_1} = 0 \}$.

(ii) Phase 2: Determine the optimal manufacturer's profit in (21). Substituting ρ, q_1, (7), and $q_2 = n_1 q_1$ into (21) yields two independent decision variables, $\{ n_2, n_1 \}$. Solve the following simultaneous equations: $\{ \frac{\partial TP_m}{\partial n_1} = 0, \frac{\partial TP_m}{\partial n_2} = 0 \}$.

(iii) Let the values of n_2 and n_1 be positive integers in the neighborhood of the values from (ii). Then, compute the total profit in (21).

(iv) The optimal value of $\{ n_2$ and $n_1 \}$ must satisfy the following conditions:

$$TP_m(n_2 - 1, n_1, \rho, q_1) \le TP_m(n_2, n_1, \rho, q_1) \ge TP_m(n_2 + 1, n_1, \rho, q_1) \quad (22)$$

and

$$TP_m(n_2, n_1 - 1, \rho, q_1) \le TP_m(n_2, n_1, \rho, q_1) \ge TP_m(n_2, n_1 + 1, \rho, q_1) \quad (23)$$

The solution procedure listed in this section is the sufficient condition for the extreme value of the total profit function in each scenario from 1-4. The necessary condition for assuring optimal total profit is for the Hessian matrix of the total profit function in Scenario 1-4 to be negative definite (ND) or negative semi-definite (NSD).

5. Numerical Example

An example of the glass bottle (component) of a soft drink (finished product) was used to demonstrate the model. The glass bottle can be supplied using both new and recycled components from the component supplier or the consumer market, respectively. The numerical data were set as follows: $\alpha_1 = 300000$, $\beta_1 = 4000.0$, H_r =6.0000, O_r =200.00, w =20.000, p =300000, H_1 =4.0000, s =2000.0, m =8.0000, H_2 =3.0000, $O_2 = 8.0000$, $\alpha_2 = 0.1500$,

$\beta_2 = 0.17500$ and H_3=1.4000. The computational results were presented in the following four Scenarios:

Scenario 1: Recycling and integration

The optimal solution in Scenario 1 is shown in Table 1.

Table 1. Optimal solution in Scenario 1 (n_2 and n_1 are real values)

n_2	n_1	u	H_r	q_1	q_2	TP_r (10^3)	TP_m (10^3)	TP (10^3)
2.605	4.614	3.552	39.851	714.31	3295.8	2,772.6	2,129.3	4,901.9

In Scenario 1, by using the solution procedure stated in Section 3, the solution set is { n_2 =2.605, n_1 =4.614, u =3.552, ρ =39.851, q_2 =3295.8, TP =4,901.9(10^3)} when n_2 and n_1 are real values. Because n_2 and n_1 are positive integers, the actual solution set is shown in Table 2 as { n_2 =3, n_1 =5, u =3.549, ρ =39.848, q_2 =3401.6, TP =4,901.9(10^3)}.

Table 2. Optimal solution in Scenario 1 (n_2 and n_1 are positive integers)

n_2	n_1	u	ρ	q_1	q_2	TP_r (10^3)	TP_m (10^3)	TP (10^3)
3	4	3.548	39.856	794.46	3177.8	2,772.7	2,129.0	4,901.8
*3	*5	3.549	39.848	680.32	3401.6	2,772.4	2,129.5	*4,901.9
3	6	3.549	39.843	599.49	3596.9	2,771.9	2,129.6	4,901.5
4	5	3.541	39.847	690.58	3452.9	2,772.4	2,129.4	4,901.8
2	5	3.560	39.850	666.31	3331.5	2,772.5	2,129. 3	4,901.8

Remark: *****optimal solution

Scenario 2: Recycling without integration

Based on the solution procedure described in Section 3, the optimal solution set { q_1 =2706.2, ρ =47.537, n_2 =2, n_1 =4, u =3.549} is shown in Table 3.

Table 3. Exploration of the optimal solution in Scenario 2

Phase 1 decision-making	Phase 2 decision-making	$TP_r + TP_m$

q_1	ρ	TP_r (10^3)	n_2	n_1	u	TP_m (10^3)	TP (10^3)
2706.2	47.537	3,008.8	2	3	3.546	1,655.1	4,663.9
2706.2	47.537	3,008.8	*2	*4	3.549	*1,657.6	*4,666.3
2706.2	47.537	3,008.8	2	5	3.550	1,657.3	4,666.0
2706.2	47.537	3,008.8	3	4	3.538	1,657.5	4,666.2
2706.2	47.537	3,008.8	1	4	3.569	1,657.3	4,666.0

Scenario 3: Without recycling but with integration

The results of exploring the optimal solution by using the solution procedure in Section 3 are shown in Table 4. Regarding Scenario 3, the optimal solution set {n_2=12, n_1=5, ρ=41.572, q_1=3102.0, TP=\$4,435.6(10^3)} is shown in Table 4.

Table 4. Exploration of the optimal solution in Scenario 3

n_2	n_1	ρ	q_1	q_2	TP_r (10^3)	TP_m (10^3)	TP (10^3)
11	5	41.572	3093.4	15467.1	2,866.5	1,569.1	4,435,588
*12	*5	41.572	3102.0	15510.0	2,866.5	1,569.1	*4,435,597
13	5	41.572	3109.9	15549.5	2,866.5	1,569.1	4,435,595
12	4	41.579	3606. 7	14426.8	2,866.5	1,568.8	4,435,303
12	6	41.567	2742.4	16454.3	2,866.2	1,569.2	4,435,405

Scenario 4: Without recycling or integration

The results of the optimal solution are shown in Table 5.

Table 5. The optimal solution in Scenario 4

Phase 1 decision-making			Phase 2 decision-making				$TP_r + TP_m$
q_1	ρ	TP	n_2	n_1	TP_m	q_2	TP
2706.2	47.537	4,293,649	10	5	1,284,892	13530.963	4,293,649
2706.2	47.537	*4,293,652	*11	*5	*1,284,895	13530.963	*4,293,652
2706.2	47.537	4,293,643	12	5	1,284,886	13530.963	4,293,643

2706.2	47.537	4,292,979	11	4	1,284,222	10824.771	4,292,979
2706.2	47.537	4,292,911	11	6	1,284,154	16237.156	4,292,911

Table 6. Comparison of computational results for Scenario 1-4

Scenario i	Scenario 1: w/ recycling w/ integration	Scenario 2: w/ recycling w/o integration	Scenario 3: w/o recycling w/ integration	Scenario 4: w/o recycling w/o integration
n_2	3	2	12	11
n_1	5	4	5	5
ρ	39.848	47.537	41.572	47.537
u	3.549	3.549	----------	----------
q_2	3401.6	2478.1	15,510	13,531
q_3	11455	8346.7	----------	-----------
q_1	2971.3	2,706.2	3,102.0	2,706.2
TP_r (10^3)	2,772.4	3,008.8	2,866.5	3,008.8
TP_m (10^3)	2,129.5	1,657.6	1,569.1	1,284.9
TP (10^3)	4,901.9	4,666.3	4,435.6	4,293.7
TP ratio	114.17%	108.68%	103.31%	100.00%

Remark: w/: with considering; w/o: without considering; TP ratio: based on Scenario 4

The following observations were based on the computational results for scenario 1-4 listed in Table 6:

(i) When integration is considered, the total profit in Scenario 3 increases 3.31% more than that in Scenario 4, and the total profit in Scenario 1 increases approximately 5.05% more than that in Scenario 2.

(ii) When recycling is considered, the total profit in Scenario 2 increases 8.68% more than that in Scenario 4, and the total profit in Scenario 1 increases approximately 10.51% more than that in Scenario 3.

(iii) When integration and recycling are considered simultaneously, the total profit in Scenario 1 increases 14.17% more than that in Scenario 4.

(iv) When integration is considered, the retailer's lot size is more than that without integration.

(v) When recycling and/or integration are considered, the manufacturer has more benefit than the retailer. Therefore, through negotiation, the manufacturer may offer a quantity discount or credit terms to convince the retailer to join this long-term, win-win partnership.

(vi) When integration and recycling are considered for Scenario 1, the selling price is reduced 4.15% more than that in Scenario 3.

(vii) When integration is considered, the selling price of considering recycling decreases 16.17% (Scenario 1) compared to that without considering recycling (Scenario 2).

(viii) The additional benefit incurred from recycling and integration can be shared by all players, including the end consumer, through the price discount offered by the manufacturer to the retailer. This win-win strategy ensures a long-run partnership in the supply chain.

6. Sensitivity Analysis

Sensitivity analysis was conducted when one of the parameters in the set { α_1, β_1, α_2, β_2, u, ρ, m } was changed by fixed rates ($\pm 10\%, \pm 20\%$ and $\pm 30\%$).

Table 7. Sensitivity analysis of the demand scale parameter

α_1	n_2	n_1	u	ρ	q_1	q_2	TP (10^3)
210000	2	4	3.544	28.642	2696.2	2478. 6	2,246.7
240000	2	4	3.550	32.379	2944.4	2694.9	3,018.8
270000	2	4	3.555	36.117	3186.7	2905.6	3,903.9
{300000}	3	5	3.549	39.848	2971.2	3401.6	4,901.9
330000	3	5	3.553	43.588	3192.7	3643.2	6,012.8
360000	3	5	3.557	47.328	3417.8	3888.2	7,236.6
390000	4	6	3.554	51.060	3282.3	4491.3	8,573.4

Remark: { } Base column

As shown in Table 7, the selling price and new-component lot size increase with the increase in the demand scale parameter, whereas the return price is comparatively less sensitive to the change in the demand scale parameter.

Table 8. Sensitivity analysis of the demand price-sensitive parameter

β_1	n_2	n_1	u	ρ	q_1	q_2	TP (10^3)
2800	3	5	3.550	55.918	3012.6	3446.8	7,306.0
3200	3	5	3.549	49.222	2998.8	3431.7	6,303.7
3600	3	5	3.549	44.014	2985.0	3416.7	5,524.7
{4000}	3	5	3.549	39.848	2971.2	3401.6	4,901.9
4400	3	5	3.548	36.440	2957.5	3386.5	4,392.7
4800	3	5	3.548	33.600	2943.7	3371.5	3,968.8
5200	3	5	3.548	31.196	2929.9	3356.4	3,610.4

As shown in Table 8, when the demand price-sensitive parameter increases, the selling price decreases to promote the sales volume and counteracts the demand shrinkage in the low season. The return price is comparatively less sensitive to the demand price-sensitive parameter.

Table 9. Sensitivity analysis of the return scale parameter

α_2	n_2	n_1	u	ρ	q_1	q_2	TP (10^3)
0.105	3	5	3.680	39.947	2963.3	3719.5	4,874.4
0.120	3	5	3.636	39.914	2965.9	3613.4	4,883.5
0.135	3	5	3.593	39.882	2968.5	3507.4	4,892.6
{0.150}	3	5	3.549	39.848	2971.2	3401.6	4,901.9
0.165	3	5	3.505	39.815	2974.1	3295.9	4,911.2
0.180	3	5	3.461	39.781	2977.1	3190. 5	4,920.7
0.195	2	5	3.429	39.748	2944.0	3016.6	4,930.2

As shown in Table 9, when the return scale parameter increases, the return price decreases, and the new-component lot-size (i.e., q_2) decreases correspondingly.

Table 10. Sensitivity analysis of the return price-sensitive parameter

β_2	n_2	n_1	u	ρ	q_1	q_2	TP (10^3)
0.1225	5	5	3.368	40.264	2994.2	6548.7	4,787.6
0.1400	4	5	3.446	40.126	2977.5	5472.7	4,825.4
0.1575	4	5	3.501	39.987	2984.8	4455.6	4,863.5
{0.1750}	3	5	3.549	39.848	2971.2	3401.6	4,901.9
0.1925	2	5	3.588	39.709	2956.3	2354.5	4,940.5
0.2100	1	5	3.626	39.570	2935.8	1300.6	4,979.4
0.2275	infeasible (∵ return rate is not greater than one)						

As shown in Table 10, when the return price-sensitive parameter increases, the return price increases to promote the return volume, and the new-component lot-size (i.e., q_2) and number of deliveries (i.e., n_2) decrease accordingly. The selling price also decreases as the component decreases.

Table 11. Sensitivity analysis of the return price

u	n_2	n_1	ρ	q_1	q_3	q_2	TP (10^3)
0.6	9	5	40.625	3073.0	3918.0	11446.8	4,690.0
1.2	8	5	40.343	3059.4	5507.0	9790.1	4,766.8
1.8	6	5	40.125	3029.6	7043.8	8104.2	4,826.7
2.4	5	5	39.970	3010.8	8580.8	6473.2	4,869.3
3.0	4	5	39.877	2991.0	10094.6	4860.4	4,894.4
*3.549	3	5	39.848	2971.3	11454.7	3401.6	*4,901.9
3.6	3	5	39.848	2972.4	11592.3	3269.6	4,901.8
4.2	1	5	39.885	2901.6	12839.8	1668.4	4,891.5
4.8	1	5	39.982	3249.6	16085.7	162.48	4,858.0

≥ 4.86	infeasible (∵ return rate is not greater than one)

As shown in Table 11 and Figure 3, when the return price decreases, the return volume also decreases. When the return price increases, the return volume also increases, resulting in the total profit increasing due to lower component costs. When the return price reaches \$3.5488, the total profit has a maximal value of \$4,901.9(10^3). This study therefore found the optimal return value that maximizes the total profit.

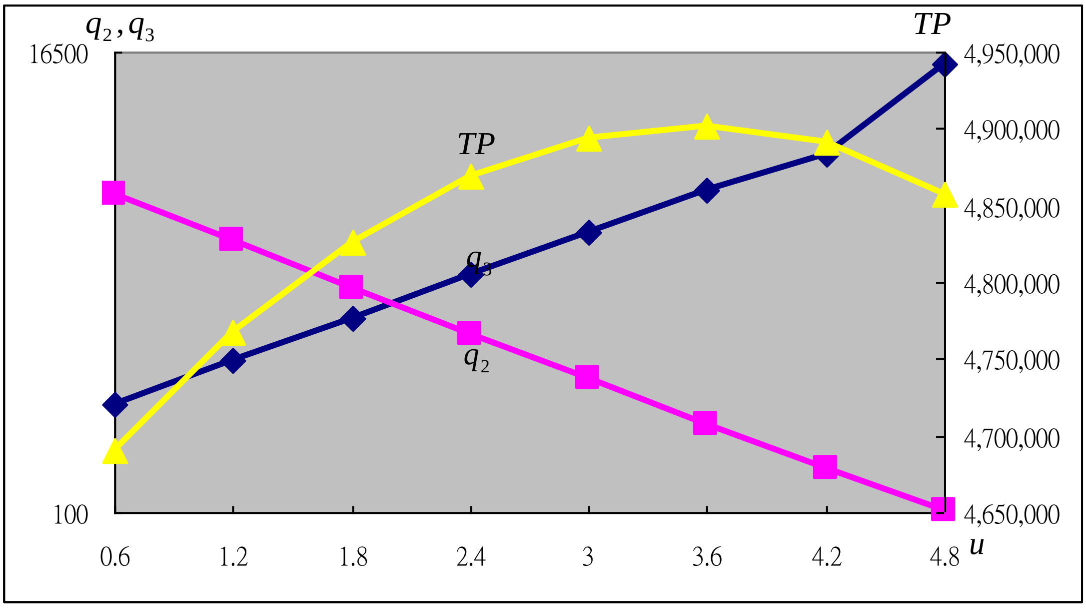

Figure 3. Relation between u and $\{q_2, q_3, TP\}$

Table 12. Sensitivity analysis of the selling price

ρ	n_2	n_1	u	q_1	q_2	$TP\,(10^3)$
24.0	4	7	3.559	3242.1	5153.8	3,900.5
32.0	3	6	3.558	3049.6	4159.7	4,656.2
*39.848	3	5	3.549	2971. 3	3401.6	*4,901.9
40.0	3	5	3.549	2962.4	3391.9	4,901.8
48.0	2	4	3.549	2903.9	2659.7	4,637.3
56.0	2	3	3.535	2938.6	2039.5	3,862.8

64.0	1	3	3.532	2166.0	1506.7	2,579.3
72.0	1	2	3.470	1553.7	754.2	790.37

From Table 12 and Figure 4, when the selling price increases, the sales revenue and total profit increase simultaneously. When the selling price reaches $39.848, the total profit reaches a maximal value $4,901.9($10^3$). If the selling price continues to increase, the total profit will decrease. Thus, this study found the optimal selling price that maximizes the total profit.

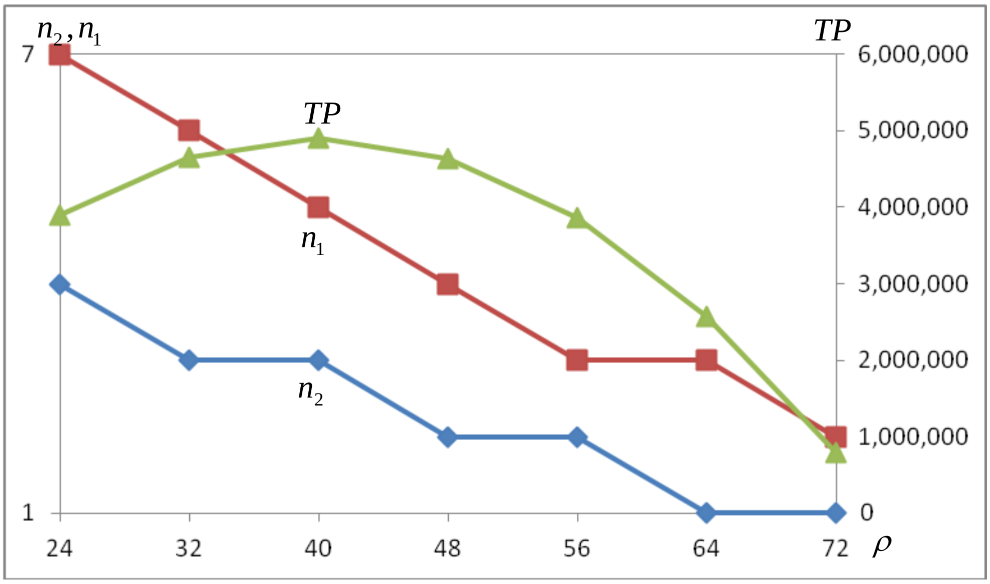

Figure 4. Relation between selling price and $\{n_2, n_1, TP\}$

Table 13. Sensitivity analysis of the purchase cost

m	n_2	n_1	u	ρ	q_1	q_3	q_2	$TP\,(10^3)$
5	6	5	2.052	39.314	3072.0	7818.8	7541. 4	5,054.8
6	5	5	2.551	39.536	3034.4	9049.3	6122.5	4,991.1
7	4	5	3.050	39.714	3000.6	10259.3	4743.5	4,940.2
8	3	5	3.549	39.848	2971.2	11454.6	3401.6	4,901.9
9	2	5	4.043	39.939	2948.3	12642.3	2099.0	4,876.0
10	1	5	4.513	39.985	2938.5	13808.6	883.9	4,862.2

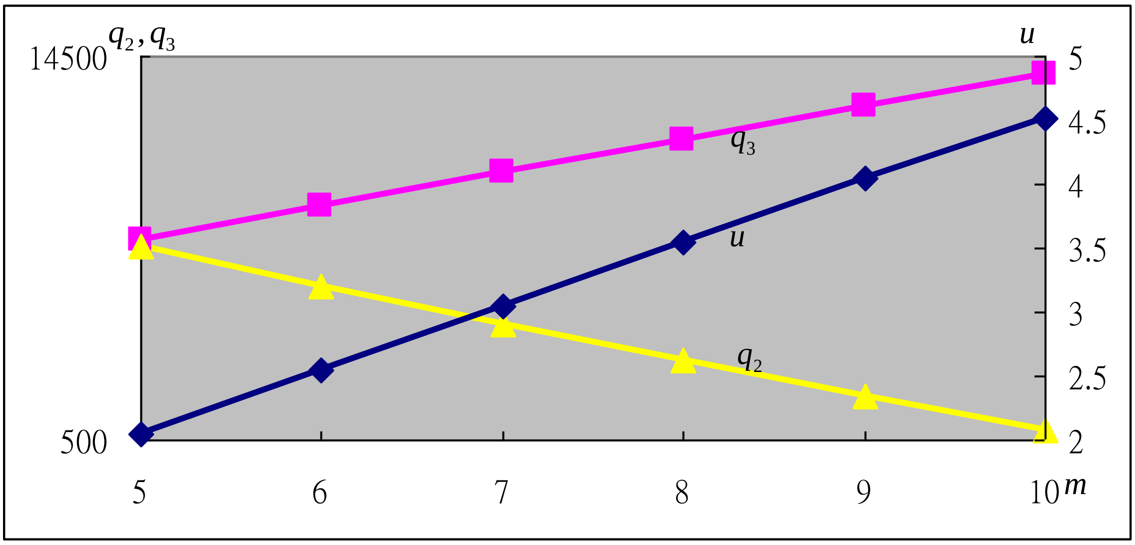

Figure 5. Relationship between m and $\{q_2, q_3\}$

From Table 13 and Figure 5, when the new-component purchase cost increases, the new-component lot-size decreases. The recycled return price and volume increase as a result.

7. Concluding Remarks

Four mathematical models considering and without considering recycling/integration were developed and compared. From the analysis, the following observations were made. When integration is considered, the total profit increases by 3.31%–5.05%. When recycling is considered, the total profit increases by 8.68%–10.51%. When recycling and integration are considered simultaneously, the total profit increases by 14.17%. Since most additional profit resulting from recycling and integration benefits only the manufacturer, the manufacturer should offer credit terms or quantity discounts to the retailer to encourage integration and benefit sharing. When either recycling or integration is considered, the selling price can be decreased by 4.15% or 16.17% respectively. In other words, recycling or integration is also beneficial to the end consumer, in addition to the joint manufacturer-retailer (Table 6). It is appropriate to raise the selling price when the demand scale parameter increases and/or the price-sensitive parameter decreases, and vice versa. It is appropriate to increase the return price when the return scale parameter decreases and/or the return price-sensitive parameter decreases, and vice versa. When the return price is too high (or too low), the lot size of recycled (or new) components becomes larger (or smaller), and it therefore is not beneficial to the total profit. An optimal return price maximizes the total profits of the manufacturer and retailer (Figure 3). A selling price that is too low (or too high) benefits the manufacturer (or the retailer) by resulting in higher production volume (or sales revenue). An optimal selling price maximizes the total profits of the manufacturer and retailer (Figure 4). When the purchase cost of new components increases, the lot size of the new components decreases, and the return price increases to raise the return volume (Figure 5). This study assumed a single manufacturer and single retailer. Cases that include multiple suppliers, collectors, manufacturers and retailers may be considered in further research.

References

Banerjee, A., 1986. A joint economic lot size for purchaser and vendor, Decision Science 17 (3) 292-311.

Clark, A.J., Scarf, H., 1960. Optimal policies for a multi-echelon inventory problem, Management Science 6(4), 475-490.

Choi, D.W., Hwang, H., Koh, S.G., 2007. A generalized ordering and recovery policy for reusable items, European Journal of Operational Research 182 (2), 764-74.

Feng, Y., Viswanathan, S., 2011. A new lot-sizing heuristic for manufacturing systems with product recovery, International Journal of Production Economics 133, 432-438.

Gaur, J., Amini, M. Rao, A.K., 2017. Closed-loop supply chain configuration for new and reconditioned products: An integrated optimization model. Omega 66, 212-223.

Georgiadis, P., Athanasiou, E., 2013. Flexible long-term capacity planning in closed-loop supply chain with remanufacturing, European Journal of Operational Research 225, 44-58.

Goyal, S.K., 1988. A joint economic lot size model for purchaser and vendor: A comment, Decision Sciences 19 (1), 236-241.

Guide Jr., V.D.R., Teunter, R.H., Van Wassenhove, L.N., 2003. Matching demand and supply to maximize profits from remanufacturing, Manufacturing and Service Operations Management 5(4), 303-16.

Hong, I., Chen, P., & Yu, H. (2016). The effects of government subsidies on decentralised reverse supply chains. *International Journal of Production Research, 54(13),* 3962-3977.

Jaber, M.Y. Saadany, A.M.A.E., 2009. The production, remanufacture and waste disposal model with lost sales, International Journal of Production Economics 120, 115-124.

Kabirian, A., 2012. The economic production and pricing model with lot-size-dependent production cost, Journal of Global Optimization 54 (1), 1-15.

Koh, S.G., Hwang, H., Sohn, K.I., Ko, C.S., 2002. An optimal ordering and recovery policy for reusable items, Computers & Industrial Engineering 43, 59-73.

Liou, Y.C., Schaible, S., Yao, J.C., 2006. Supply Chain Inventory Management via a Stackelberg Equilibrium, Journal of Industrial and Management Optimization 2, 81-94.

Mendoza A., Ventura J.A., 2012. Analytical models for supplier selection and order quantity allocation. Applied Mathematical Modelling 36, 3826-3835.

Mota, B., Gomes, M.I., Carvalho, A., Barbosa-Povoa, A.P., 2018. Sustainable supply chains: An integrated modeling approach under uncertainty. Omega 77, 32-57.

Özceylan E, Paksoy T, Bektaş T., 2014. Modeling and optimizing the integrated problem of closed-loop supply chain network design and disassembly line balancing. Transportation Research Part E: Logistics and Transportation Review 61, 142-164.

Pazhani S.,Ventura J.A., Mendoza A., 2015. A serial inventory system with supplier selection and order quantity allocation considering transportation costs. Applied Mathematical Modelling.

Rubio, S., Corominas, A., 2008. Optimal manufacturing-remanufacturing policies in a lean production environment, Computers & Industrial Engineering 55 (1), 234-42.

Shi, J., Zhang, G., Sha, J., 2011. Optimal production and pricing policy for a closed loop system, Resource Conservation and Recycling 55, 639-647.

Teunter, R., 2004. Lot-sizing for inventory systems with product recovery, Computers & Industrial Engineering 46, 431-441.

Von Stackelberg, H., 1952. Theory of the Market Economy, Oxford University Press, New York.

Wang, Q., Li, J., Yan, H., Zhu, S.X. (2016). Optimal remanufacturing strategies in name-your-own-price auctions with limited capacity. *International Journal of Production Economics*, *181*, 113-129.

Yu, Y., Huang, G.Q., Liang, L., 2009. Stackelberg game-theoretic model for optimizing advertising, pricing and inventory policies in vendor managed inventory (VMI) production supply chains, Computers & Industrial Engineering 57, 368-382.